CONVERGING DETONATION WAVES IN REACTIVE MEDIUM USING REAL GAS MODEL

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ABSTRACT

Propagation of converging detonation waves in reactive medium using BKW-equation of state is studied, with the help of Whitham's method of characteristics. It is found that in the case of strong detonation waves ideal gas equation is just an approximation Increase of detonation pressure in ideal case is very large as compared to real gas state. Results are compared with that of earlier work.

Studies of converging detonation waves in reactive media are of immense importance, for the generation of high temperature and pressure Abarbanel (1967), Singh (1978, 81), studied converging detonation waves in solid explosives, where as Lee (1965, 1967) studied in gaseous explosives. In these papers it was assumed that mediumobeys the ideal gas law. But it is known that ideal gas law does not hold at high temperatures and pressures. In the present paper, propagation of converging detonation wave in solid explosive is studied by using BKW-(Becker-Kistiakowsky - Wilson) equation of state. Whitham's method (Whitham (1958)) of characteristics is used to find a relation between the detonation front radius and the detonation parameters.

2. Variation of detonation pressure ratio z $\left(z = \frac{p\ (\Gamma + 1)}{\rho\ d^2}\right.$, BKW-parameter x and modified a diabatic constant Γ for TNT is drawn in figures 1, 2, 3 respectively. It is found that variation of z in real gas model is quite less as compared to ideal gas model and variation of parameters x and Γ first increase and then decrease continuously. Results are compared with those of earlier papers.

Formulation of the Problem

3. Let us assume that a spherical detonation wave starts from th

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surface of the sphere of solid explosive and moves towards the cen[tre]. At any time t, let r be the distance of detonation front from the [centre]. The detonation pressure p, velocity U, particle velocity u and den[sity] ρ behind the detonation front are related by

$$\rho\,(U - u) = \rho_1 U \qquad \ldots\ldots (1)$$

$$\rho\,(U - u)^2 + p = \rho_1 U^2 \qquad \ldots\ldots (2)$$

$$\tfrac{1}{2}\,(U-u)^2 + E + \frac{p}{\rho} = \tfrac{1}{2}\,U^2 + Q \qquad \ldots\ldots (3)$$

where ρ_1, E are density and internal energy of medium ahead of deto[nation] front. Here we have neglected p_1 as compared to detonation pressure p since $p_1 \ll p$. If D is the C-J detonation velocity a[nd] Q the energy released by the explosive. We have.

$$D^2 = 2\,(\Gamma^2 - 1)\,Q \qquad \ldots\ldots (4)$$

where Γ is the modified a diabetic constant. given by

$$\Gamma = \frac{-v}{p}\left(\frac{C_p}{C_v}\right)\left(\frac{p}{v}\right)_T \qquad \ldots\ldots (5)$$

v being the specific value and C_p, C_v specific heats at constant pressure and temperature. Here it is to be noted that C_p and C_v are not constants and are function of temperature T. (Sorensen 1961)

Equation of state of explosive products is (Kamlet et al (1968)

$$pv = RT\,(1 + x\,e^{\beta x})$$

where $\ldots\ldots (6)$

$$x = \frac{K\,H\,M}{v(T+\theta)^{\frac{1}{2}}} \qquad \ldots\ldots (7)$$

K, H and B are constant of explosive, M is the molar weight of the explosive products, R is the universal gas constant.

Solving (1)-(3) with the help of (4)-(7).

we have

$$u = \left(\frac{D}{\Gamma + 1}\right)\sqrt{(2Z - 1)} \qquad \ldots\ldots (8)$$

$$U = Dz \,/\, \sqrt{(2z - 1)} \qquad \ldots\ldots (9)$$

$$\rho = \frac{\rho_1 (\Gamma + 1)\, z}{\left\{1+(\Gamma - 1)\, z\right\}} \qquad \cdots\cdots$$

$$p = \frac{\rho_1 D^2\, z}{(\Gamma + 1)} \qquad \cdots\cdots$$

$$T = \frac{M}{R}\left(\frac{D}{\Gamma+1}\right)^2 \frac{\left\{1+(\Gamma - 1)\, z\right\}}{(1 + \mu)} \qquad \cdots\cdots$$

where

$$\mu = x\, e^{\beta} \qquad \cdots\cdots$$

from relation (5) we get

$$\Gamma = \gamma\left[1 + \left(\frac{1 + \beta x}{1 + \mu}\right)\mu\right] \qquad \cdots\cdots ($$

where

$$\gamma = C_p/C_v \qquad \cdots\cdots ($$

In equations (8)-(12), we have expressed u, U, ρ , p,T interms o

and μ , where as μ and Γ are given by relations (13) and (4).

Relation (7) is an extra relation, which relates x with ρ and

Thus relations (8)-(12) along with relation (7), (13) and (14)

all the detonation parameters interms of z only. It is not pos

at this stage to solve for detonation parameters in terms of on

parameter but in the next section, we will see that these relat

are of great help in solving the complete problem. Relation (7

be expressed in a more convenient form with the help of (2) and

i.e.,

$$x = \frac{KH\, \rho_1 (\Gamma + 1)^2\, z}{D} \sqrt{\frac{R(1 + \mu)}{\left\{1+(\Gamma - 1)z\right\}^3}} \qquad \cdots ($$

4. Thus we have obtained six relations (8)-(10),(12), (14) a

(15), which relate seven unknowns U, u, ρ , T, Γ , x, z. In ord

to know the history of detonation front, one should be able to r

at least one of these parameters (say z) with the distance trav

by it. Once variation of z with distance r is known, other par

can also be determined. In the next section we will find the va

of z with the radius of detonation front.

Propagation of Spherical Detonation Wave:

5. Our aim in this section is to find the variation of streng… detonation front i.e. z as it moves towards its radius of conve… Following Singh (1978), we substitute expression for p, ρ, u fr… relations (8)-(11) in the equations of motion along the positive characteristic axis. i.e.

$$dp + \rho c \, du + \frac{2 \rho c^2 u}{u+c} \frac{dr}{r} = 0 \qquad \ldots\ldots (1$$

where

$$c = \sqrt{\frac{\Gamma p}{\rho}} \qquad \ldots\ldots (1$$

Substituting expression for p, ρ, u and c from (8) - (11) and (17… in equation (16) we get after some simplifications

$$F_1 \frac{dz}{z} + F_2 \frac{d\Gamma}{\Gamma+1} + 2 F_3 \frac{dr}{r} = 0 \qquad \ldots\ldots (18$$

where

$$F_1(\Gamma, Z) = \left[1 + z \sqrt{\frac{\Gamma}{(1+(\Gamma-1)z)(2z-1)}} \right]$$

$$F_2(\Gamma Z) = - \left[1 + \sqrt{\frac{\Gamma(2z-1)}{1+(\Gamma-1)z}} \right]$$

$$F_3(\Gamma Z) = \Gamma / \left[1 + \sqrt{\frac{\Gamma(1+(\Gamma-1)z)}{(2z-1)}} \right] \qquad \ldots\ldots (19$$

Equation (18) is a differential relation between $\frac{dz}{dr}$ and $\frac{d\Gamma}{dr}$ the coefficients of which are function of z and Γ.

6. Differential relation for Γ and x are obtained by differen-tiating (14) and (15) respectively. i.e.

$$\frac{d\Gamma}{dr} = \lambda(z) \frac{dx}{dr} \qquad \ldots\ldots (20)$$

$$\frac{z}{x} \frac{dx}{dr} \nu(x, z) = \frac{dz}{dr} \qquad \ldots\ldots (21)$$

where

$$\lambda(x) = \gamma \left[\frac{(1+Bx)^2 \; e^{Bx} + Bu(1+\mu)}{(1+\mu)^2} \right] \quad \ldots\ldots (22)$$

$$\nu(x, z) = \left[\left\{ 2 - \frac{(1+Bx)\,\mu}{1+\mu} \right\} \left\{ (1 + (\Gamma - 1)\, z \right\} - \frac{x\,\dot{\lambda}(x)(4 - (\nabla - \Gamma)\, z\}}{(\Gamma + 1)} \right] \Big/ \left[2 - (\Gamma - 1)\, z \right] \quad \ldots (23)$$

Relations (18), (20) and (21) when combined give

$$\frac{r}{2} \; \frac{dz}{dr} = -K_1 \quad \ldots . (24)$$

$$\frac{r}{2} \; \frac{dx}{dr} = -\frac{x\,k_1}{\nu z} \quad \ldots . (25)$$

where

$$K_1 = \frac{F_3}{\left[F_1 + \frac{F_2\,\lambda\,x}{\nu(\Gamma + 1)} \right]} \quad \ldots . (26)$$

Differential relations (24) and (25) give variation of z and x as the detonation wave moves along the radius r. Parameter Γ occurring in these relation is a function of x given by (14).

7. When we know z and x as a function of r, other parameters can be obtained from the jump conditions.

Numerical Calculation and Comparison of Results

8. We have integrated equations (24), (25) along with (14) with the help of computer using Range Kutta method of fourth order. Following data (Mader 1979) was used, to study the propagation of detonation waves in TNT.

ρ_1 = 1.64 g/cc

D = 6.95×10^5 cm/sec^2

p_{CJ} = 206 kbar

θ = 400°k

$K = 12.685$

$H = 13.76$

$\alpha = 0.5$

$\beta = .09585$

$T_{CJ} = 2937^{0}k$

$\Gamma_{CJ} = 2.85$

Variation of detonation pressure ratio z versus (r/r_0) is also found by ideal gas law model (Singh 1978), r_0 be initial radius of detonation front. In figure 1, we have plotted variation of z versus r/r_0 for both the cases. In figure 2 and 3 variations of x & Γ versus distance (r/r_0) are shown.

Conclusion

9. It is found that variation of detonation pressure ratio z increases much slowly in the present case. In fig. 1, variation of z versus (r/r_0) is drown for two cases i.e. for ideal gas law and for BKW-gas law. It is found that increase in z is quite large in the first case as compared to second. Figures 2 and 3 show that x and Γ first increase and then decrease as detonation wave moves from surface towards the centre. This is due to the fact that as detonation wave moves towards the centre, temperature goes on increasing but density ρ becomes constant. Thus from relation (7) it can be seen that x first increases and afterward decreases as inverse square root of temperature. Since Γ depends on x it also behaves in a similar fasion. Variations of x and Γ versus r/r_0 are shown in figure 2 and 3.

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Caption of figures

Figure 1. Comparison of pressure - distance law for BKW and ideal gas models.

Figure 2. Variation of parameter x versus distance ratio r/r_o.

Figure 3. Variation of adiabatic parameter Γ versus distance ratio r/r_o.

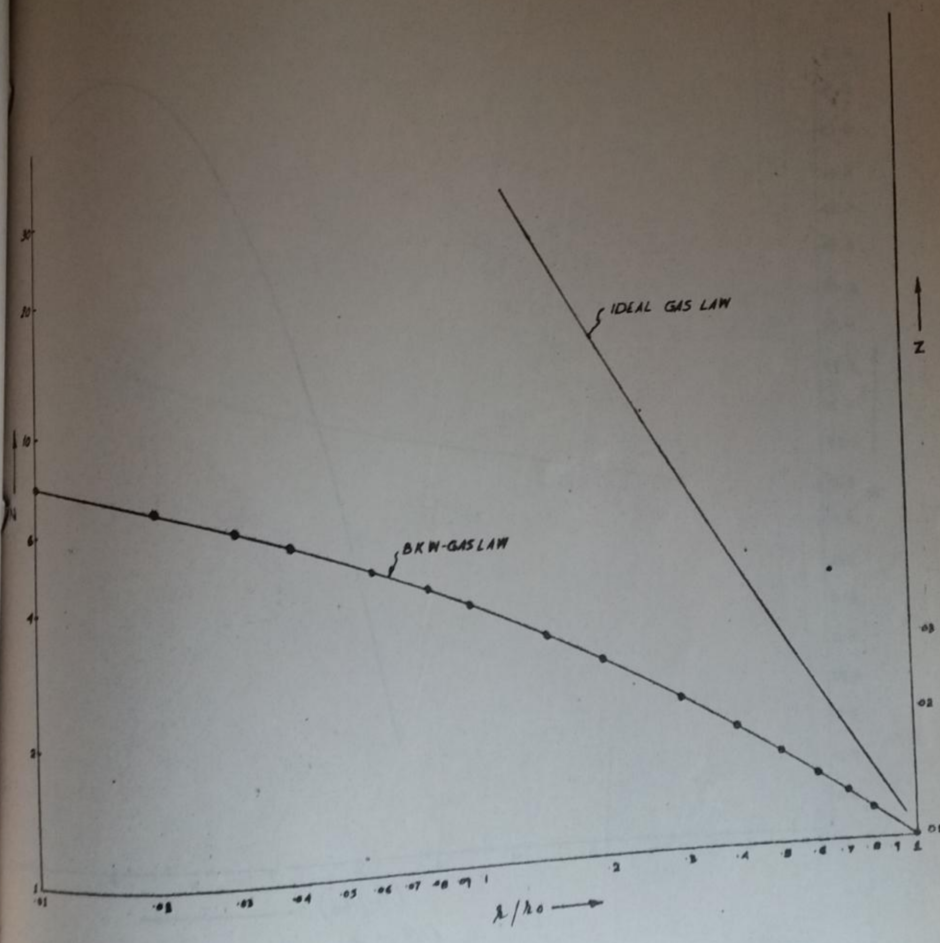


FIG. 1

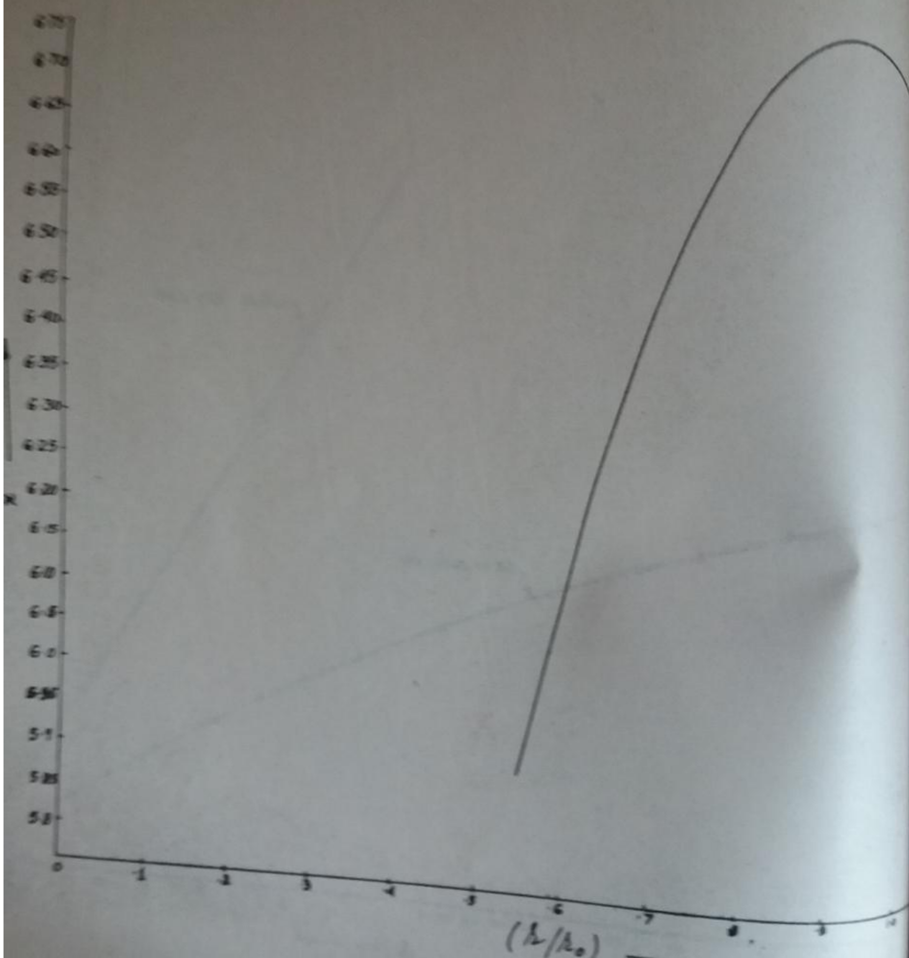
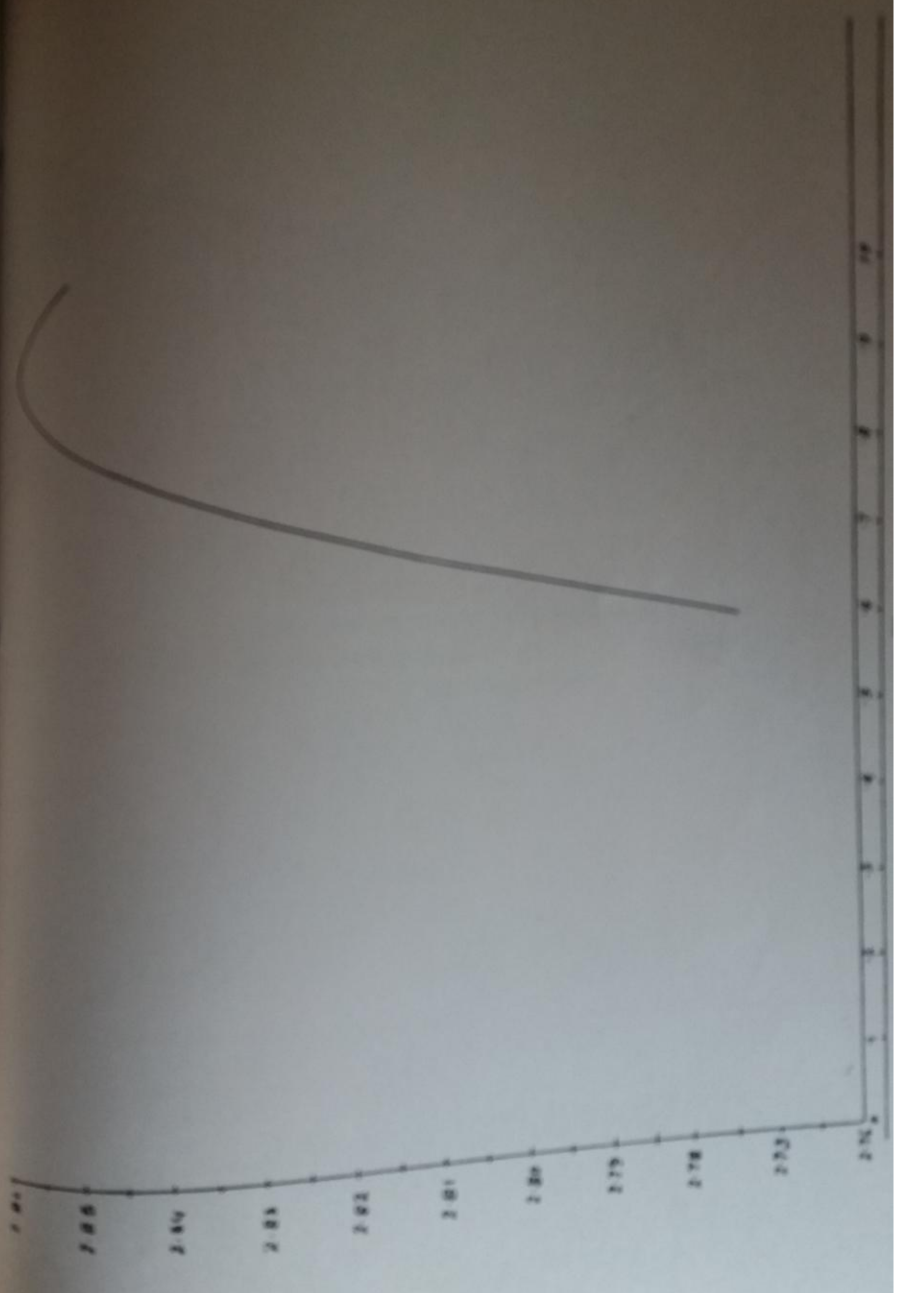


FIG :- 2



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