SHOCK WAVES IN EARTH'S ATMOSPHERE

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Abstract:—Propagation of shock waves, produced by rockets or objects moving with supersonic speed in earth's atmosphere are studied by using Whitham's method of characteristics, for the general case of ascending spherical shock waves, it is found that shock wave first decays but after reaching a specific height, its strength starts increasing. For the case of plane oblique shocks its strength increases as it moves upward. In the case of plane shocks, analytical expression for the shock velocity is found.

1·1 Introduction

Shock waves is quite common phenomena, in nature. They are produced by explosion or by the motion of objects moving with supersonic speed in the earth's atmosphere. These shock waves are generally curved. The study of propagation of curved shock waves in earth's atmosphere, which itself is nonuniform, is a quite complex problem. Singh (1969) studied propagation of plane shock waves in the earth atmosphere moving in the vertically upward direction. This case was generalised to plane oblique shocks by Singh and Kumar (1970). Later Nunziato and Walsh (1972, 1973) developed a different theory for the propagation of shock waves and while applying to earth's atmosphere, they found that their results were in quite agreement with those of Singh (1969).

In the present case we take a general case of spherical shock waves produced at height Z_E by some mean. In the section 1·2 we give basic formulation of the problem. In section 1·3, brief results of plane oblique shocks are given. In section 1·4 propagation of spherical shock waves in the earth's atmosphere is discussed by using Whitham's rule (1958).

It is shown that troposphere behaves as homogeneous atmosphere and shock decays in all the directions. But in the vertically upward direction shock wave first decays and then accelerates. Stronger is the shock, lesser is the distance through which it decays, in the vertically upward direction. Variations of the shock Mach number and shock velocity are computed for initial Mach number four and ten in the vertically upward direction and four only in the horizontal and vertically down-ward direction. It is concluded that the nonuniformities of the atmosphere has destabilizing effect on the shock front moving in the vertically upward direction.

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1·2 Basic Formulations

Let us assume that a spherical shock is produced at a height Z_E from the earth's surface by an explosion. If the point of explosion is taken as origin, let the position vector R_o denotes shock position, which makes an angle θ with the vertically upward direction. θ increases in clockwise direction. Throughout this paper with subscripts zero will denote values in undisturbed state and subscript 's' will denote values at the earth's surface. Let p, ρ, T, u, g be the pressure, density, temperature, fluid velocity, and acceleration due to gravity at the height z from the surface of the earth. Here onward by shock strength we imply the shock Mach number.

Let r_0, t_0, u_0, a_0 be the radial distance, time, shock velocity and earth's radius. Then we can define the dimensionless parameters as

$$u = \sqrt{r}\, u_o / c_s, \quad p = p_o / p_s, \quad \rho = \rho_o / \rho_s, \quad T = T_o / T_s,$$

$$r = r_o / Z_E, \quad a = a_o / Z_E, \quad t = \frac{c_s\, t_o}{\sqrt{r}\, Z_E}, \quad g = g_o / g_s,$$

$$U = \sqrt{r}\, U_o / c_s, \quad R = R_o / Z_E, \quad Z = Z_o / Z_E, \qquad \text{.. (1·2·1)}$$

where c_s is the sound velocity at the surface of the earth.

Thus the equations of motion in terms of non-dimensional parameters are

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial r} + u \frac{\partial \rho}{\partial r} + \frac{2\, \rho u}{r} = 0$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} + k \left(\frac{a}{a + 1 + r \cos \theta} \right)^2 \cos \theta = 0$$

$$\frac{d S}{d t} = 0 \qquad \text{.. (1·2·2)}$$

S is entropy and dimensionless parameter k is given by

$$k = \alpha\, Z_E \qquad \text{.. (1·2·3)}$$

and $\alpha = g_s\, \rho_s / p_s$ (Singh 1969).

Equation of state is

$$p = \rho^{\gamma} \qquad \text{.. (1·2·4)}$$

In equilibrium, second of (1·2·2) gives

$$\frac{1}{\rho}\,\frac{dp}{dr} = -\,k\left(\frac{a}{a+1+r\cos\theta}\right)^{2}\cos\theta \qquad \ldots \quad (1{\cdot}2{\cdot}5)$$

using (1·2·4) in (1·2·5) one sets

$$\rho = \left[1 - \frac{r-1}{r}\,\frac{k\,a\,(1+r\cos\theta)}{(a+1+r\cos\theta)}\right]^{\frac{1}{r-1}} \qquad \ldots \quad (1{\cdot}2{\cdot}6)$$

$$p = \left[1 - \frac{r-1}{r}\,\frac{k\,a\,(1+r\cos\theta)}{(a+1+r\cos\theta)}\right]^{\frac{r}{r-1}}$$

and from gas equation

$$T = \left[1 - \frac{r-1}{r}\,\frac{k\,a\,(1+r\cos\theta)}{(a+1+r\cos\theta)}\right] \qquad \ldots \quad (1{\cdot}2{\cdot}7)$$

Equation (1·2·6) and (1·2·7) gives the variations of density, pressure and temperature at any point whose radial co-ordidates are (r, θ). If we take $Z_{\text{M}} = 0$, these equation reduce to equations (14) of Singh and Kumar (1970).

1.3 Discussion of the Problem

Let us assume that at height $z = 1$, a spherical shock wave is produced by an explosion which moves in all the direction. At time t, let R is the position of the shock front, making an angle θ with the vertical upward direction. We take point of explosion as the origin. If quantity with subscript 2 denote values behind the shock front and those without any subscript in front of the shock front, then the jump in fluid parameters is given by

$$\begin{aligned} u_2\,(R, t) &= (2c/(r+1))\,(M - M^{-1}) \\ p_2\,(R, t) &= \frac{r\,p}{r+1}\,f(M) \\ \rho_2\,(R, t) &= (r+1)\,\rho\,M^2/g\,(M) \end{aligned} \qquad \ldots \quad (1{\cdot}3{\cdot}1)$$

where

$$\begin{aligned} f(M) &= \left\{2\,M^2 - \frac{r-1}{r}\right\} \\ g(M) &= \{2 + (r-1)\,M^2\} \\ M &= U/(r\,p/\rho)^{1/2} \end{aligned} \qquad \ldots \quad (1{\cdot}3{\cdot}2)$$

Equation (1-3-1) give the relations between u_2, p_2, ρ_2 and U. To solve these equations we require one extra relation which is obtained by the use of Whitham's Rule (Witham 1958). Equations of motion along the positive characteristic axis at $r=\mathrm{R}$ is (Singh and Kumar 1790),

$$d\, p_2 + \rho_2\, c_2\, du_2 + \frac{p_2\, c_2}{u_2 + c_2}\left[k\left(\frac{a}{a+1+\mathrm{R}\cos\theta}\right)^2 \cos\theta + \frac{2\, c_2\, u_2}{\mathrm{R}}\right] d\,\mathrm{R} = 0 \qquad \text{.. (1-3-3)}$$

If we substitute the expressions for p_2, u_2, c_2 from (1-3-1) and (1-3-2) in (1-3-3) and after some simplifications we get

$$2\left\{2\mathrm{M}^2 + (\mathrm{M}^2+1)\, h(\mathrm{M})\right\}\frac{d\mathrm{M}}{\mathrm{M}} + \left\{f(\mathrm{M}) + (\mathrm{M}^2-1)\, h(\mathrm{M})\right\}\frac{dp}{p}$$
$$-(\mathrm{M}^2-1)\, h(\mathrm{M})\, d\rho/\rho + [\, h(\mathrm{M})\, \mathrm{M}^2 (r+1)^2 / c^2 \{2(\mathrm{M}^2-1) + \sqrt{rfg}\}\,]$$
$$\left[k\left(\frac{a}{a+1+\mathrm{R}\cos\theta}\right)^2 \cos\theta + \frac{uc^2\sqrt{rfg}\,(\mathrm{M}^2-1)}{\mathrm{M}^2 (r+1)^2\, \mathrm{R}}\right] d\,\mathrm{R} = 0 \qquad \text{.. (1-3-4)}$$

where $h(\mathrm{M}) = \{\, r\, f(\mathrm{M}) / g(\mathrm{M})\,\}^{1/2}$ .. (1-3-5)

putting the values of dp/p and $d\rho/\rho$ from (1-2-6) in (1-3-4) one gets

$$\frac{2}{\mathrm{M}}\frac{d\mathrm{M}}{d\mathrm{R}} = \mathrm{K}_3(\mathrm{M})\,\frac{k\left(\dfrac{a}{a+1+\mathrm{R}\cos\theta}\right)^2 \cos\theta}{\left[1 - \dfrac{r-1}{r}\,\dfrac{k\,a\,(1+\mathrm{R}\cos\theta)}{(a+1+\mathrm{R}\cos\theta)}\right]} - \frac{\mathrm{K}_4(\mathrm{M})}{\mathrm{R}} \qquad \text{.. (1-3-6)}$$

which is a differential equation between M and R.

$\mathrm{K}_3(\mathrm{M})$ and $\mathrm{K}_4(\mathrm{M})$ are given as :—

$$\mathrm{K}_3(\mathrm{M}) = \mathrm{K}_1(\mathrm{M}) + \frac{r-1}{2r}\,\mathrm{K}_2(\mathrm{M})$$

$$\mathrm{K}_1(\mathrm{M}) = \left\{ f(\mathrm{M}) - \frac{(r+1)^2/r\ \mathrm{M}^2\, h(\mathrm{M})}{\{2(\mathrm{M}^2-1) + \sqrt{rfg}\}}\right\} \Big/ \left\{(\mathrm{M}^2+1)\, h(\mathrm{M}) + 2\,\mathrm{M}^2\right\}$$

$$\mathrm{K}_2(\mathrm{M}) = 2(\mathrm{M}^2-1)\, h(\mathrm{M}) / \{(\mathrm{M}^2+1)\, h(\mathrm{M}) + 2\,\mathrm{M}^2\}$$

$$\mathrm{K}_4(\mathrm{M}) = \frac{4\, r\, f(\mathrm{M})\, (\mathrm{M}^2-1)}{\{2(\mathrm{M}^2-1) + \sqrt{rfg}\}\,\{2\,\mathrm{M}^2 + (\mathrm{M}^2+1)\, h(\mathrm{M})\}} \qquad \text{.. (1-3-7)}$$

Equation (1.3.7) is the modified from of equation (15) given by Singh and Kumar (1970).

It gives the variation of the Mach number as the shock travels along any direction (R, θ). Once we know M, other parameters can be calculated from (1.3.1). Thus we know the variation of all the parameters, as the shock front propagates.

This equation can not be integrated analytically. But for the case of plane shocks, this is integrated analytically in the next section.

For spherical shocks it is integrated numerically and results are shown in the Table I.

1.4 Comparison with the Case of Plane Shock Waves

In the case of plane shock waves, which starts from the surface of the earth in the direction θ, equation (1.3.6) becomes

$$\frac{2}{M}\,\frac{dM}{dR} = K_2(M)\,\frac{\left(\dfrac{a}{a+R\cos\theta}\right)^2\cos\theta}{\left[1-\dfrac{r-1}{r}\,\dfrac{aR\cos\theta}{a+R\cos\theta}\right]} \tag{1.4.1}$$

where in this case distance is non-dimensional by the parameter α, (Singh and Kumar 1970).

It is shown by Singh and Kumar (1970) that function K_2 (M) can be taken as slowly varying function of M as M becomes greater that three. Thus equation (1.4.1) is integrated to

$$M = M_s\left[1-\frac{r-1}{r}\,\frac{aR}{a+R\cos\theta}\right]^{-\frac{r}{r-1}\frac{K_2}{2}} \tag{1.4.2}$$

where M_s is the initial value of the Mach number. Thus shock velocity is given as

$$U = \sqrt{r}\,M_s\left[1-\frac{r-1}{r}\,\frac{aR\cos\theta}{a+R\cos\theta}\right]^{-\left(\frac{rK_2}{r-1}-1\right)/2}$$

Thus we see that for the case (1.4.3) of plane shock waves, one can get analytical relation for shock velocity, whereas for spherical shocks, numerical integration is required.

1·5. Discussion of the Problem

Following data is used for numerical computation of the problem :—

$Z_E = 5 \times 10^3$ m.

$g_s = 9{\cdot}81$ m/sec^2.

$p_s = 10^6$ dynes/cm^2

$\rho_s = 1{\cdot}3 \times 10^{-3}$ gm/cm^3

$r = 1{\cdot}4$

Therefore

$k = 0{\cdot}63765$

$\alpha = 1275 \times 10^{-7}\ m^{-1}$

For the case of plane shock waves, strength (Mach number) of shock increases as shock propagates vertically upwards. But for spherical shock waves, it is seen that strength of shock first decreases and after reaching some specific height, starts increasing. This specific height depends on the initial strength of shock waves. In the horizontal direction, plane shock remains constant whereas spherical shock decays. This is due to the divergence term in the momentum equation.

It is concluded that decrease in density of the atmosphere has destabilizing effect on the shock wave, irrespective of its shape.

Results of numerical computation for spherical shocks are shown in the Table.

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NR – What is the physical argument for the shock wave tendencies later. Does Whitham's theory predict it.

– Whitham's rule predicts that spherical shock wave will decay as it moves away from the point of explosion and ultimately it will become sound wave. This is because of energy of explosion per unit volume which continuously decreases as the shock strength again starts increasing. This is because in – hamogeneities at larger height become predominant as compared to divergence terms.

Table 1 — Variation of M

R	$\theta = 0$	$\theta = 0$	$\theta = \pi/2$	$\theta = \pi$
	Minitial = 4	Minitial = 10	Mini = 4	Mini = 4
·01	4·000000	10·000000	4·000000	4·000000
0·1	1·857604	4·223126	1·831785	1·808930
0·2	1·607809	3·330889	1·574904	1·547044
0·3	1·512793	2·937989	1·473258	1·441313
0·4	1·459927	2·708578	1·413705	1·378560
0·5	1·431743	2·576144	7·380296	1·342829
0·6	1·405251	2·460941	1·346958	1·306815
0·7	1·394335	2·403059	1·330775	1·289311
0·8	1·378008	2·334594	1·306977	1·263108
0·9	1·375032	2·312155	1·298889	1·254719
1·0	1·364022	2·268930	1·282294	1·232937
1·2	1·359339	2·223627	.. ..	.. ..
1·4	1·358637	2·243582	.. ..	.. ..
1·6	1·409071	.. ..	.. ..	.. ..

Table 2 — Shock Velocity U

R	$\theta = 0$	$\theta = \pi/2$	$\theta = \pi$
·01	4·274889	4·279715	4·285490
0·1	1·965485	1·959880	1·956805
0·2	1·678644	1·685035	1·691665
0·3	1·563675	1·576281	1·592658
0·4	1·491000	1·512564	1·539184
0·5	1·444262	1·476818	1·514524
0·6	1·399752	1·441149	1·488607
0·7	1·370815	1·423835	1·483005
0·8	1·336906	1·398373	1·466785
0·9	1·315727	1·389719	1·462673
1·0	1·287199	1·371964	1·458564