© *Proc. Indian natn. Sci. Acad.*, **47**, A, No. 4, 1981, pp. 435–438.
Printed in India.
Fluid Dynamics

CONVERGING SHOCKS WITH HEAT ADDITION

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(*Received* 24 July 1979*)

Propagation of converging detonation waves is discussed. Chisnell's method is modified for explosive media. Using Conger's method of folded coordinates, it is found that detonation wave becomes stronger and stronger as the wave front approaches the centre.

Keywords: Converging Shocks; Imploding Detonation Waves; Shocks with Heat Addition; Converging Detonation Waves.

(§1) Introduction

Propagation of shock waves in a medium with chemical reaction behind the shock has been treated by Abarbanel (1967) and Conger (1968) by using characteristics method and Singh (1979) by using Chisnell's technique. It is assumed by Abarbanel that the spherical symmetry of the front is maintained as it moves towards the point of convergence. This gives an infinite wave velocity near the point of convergence. Conger however having this reason in mind, imagined that the front becomes warped near the centre. He used a transformation which changes a spherical front into a star shaped front. Conger deduces that the front velocity is almost constant as it converges. This is perhaps due to the Chapman-Jouguet condition assumption, which he has made. We discuss this in section 2.

We have modified Chisnell's teachique (1957) of flow in a Channel with variable cross sectional area, for the case of explosives. Using conformal transformations, we have shown that the wave velocity does increase, as front converges. Following Conger, three dimensional problem is reduced to two dimensional problem. Using elliptic coordinates ξ, η, variation of pressure behind the front is found for different values of ξ and η.

(§2) Formulation of the Problem

We assume that a detonation wave is propagating in a channel of variable cross sectional area A. Initially, at the cross-sectional area A, we assume that $p = p_{CJ}$, $u = u_{CJ}$, $P = P_{CJ}$, where subscript CJ implies values at the section where Chapman-Jouguet conditions hold. Detonation velocity D is the Chapman-Jouget

*Accepted for publication on 6 November 1980.

Note: A paper (by J. H. T. Wu *et al.*, AIAAJ., 1980, pp. 47–48) appeared after the present work was communicated for publication in which it is found experimentally that the curvature of the cylindrical imploding shocks breaks down as it approaches the axis of the convergence.

(CJ) *Detonation Velocity*

The jump conditions across the detonation front are (Abarbanel, 1967)

$$W = z/(2z - 1)^{1/2} \quad ...(1)$$

$$u = \frac{D}{(r+1)}(2z - 1)^{1/2} \quad ...(2)$$

$$\rho = \frac{\rho_0(r + 1)\, z}{1 + (r - 1)\, z} \quad ...(3)$$

$$z = (r + 1)\, p/(\rho_0 D^2) \quad ...(4)$$

$$W = U/D \quad ...(5)$$

where ρ_0, D, p, u, ρ, U are respectively density of unreacted material, *C-J* wave speed, pressure, fluid velocity, density of the reacted material and actual wave velocity.

From (1) to (5), it can be shown that

$$u + c = \frac{D}{(r + 1)}\{(2z - 1)^{1/2} + r^{1/2}(1 + \overline{r - 1}\, z)^{1/2}\} \quad ...(6)$$

Here right hand side is equal to D only when $z = 1$, i.e., when $W = 1$ or $U = D$. This situation happens only in the case of non-overdriven waves (Gruschka Wecken 1971). Conger used this condition by assuming

$$u + c = D.$$

We have however waived this condition totally. Following Chisnell (1957), we have in our case,

$$-\frac{1}{A}\frac{dA}{dz} = \frac{1 + [\{1 + (r - 1)z\}/r(2z - 1)]^{1/2}}{[\{1 + (r + 1)\, z\}/r(2z - 1)]^{1/2}} \times \left[\frac{1}{z} + \frac{r\,[\{1 + (r - 1)\, z\}/r(2z - 1)]^{1/2}}{(2z - 1)}\right] \quad ...(7)$$

which on integration gives

$$\frac{A}{A_0} = \exp(M_e - 1)\left[\left(\frac{2rM_2^2 - r + 1}{r + 1}\right)^{(r+2)/2r} \times \left\{\frac{M_2 - X}{M_2 + X} \times \frac{1 + X}{1 - X}\right\}^{X/2} \left(\frac{1 + rM_2^2}{r + 1}\right)^{-1/r}\right] \quad ...(8)$$

where

$$X = [(r - 1)/2r]^{1/2}$$

$$M_2 = \frac{U - u}{(rp/\rho)^{1/2}} = \left[\frac{1 + (r - 1)\, z}{r\,(2z - 1)}\right]^{1/2} \quad ...(9)$$

which is an analytic relation between M_2 and A/A_0.

(§3) Folded Coordinates System

Conger (1968) has assumed that as spherical front approaches the centre of convergence it becomes folded and takes the shape of the star. For transforming a circle to a star shape in two dimensions, one can use the conformal transformation,

$$\bar{w} = \bar{z}^n \quad \ldots(10)$$

where

$$\bar{w} = q + is$$

$$\bar{z} = x + iy$$

$$q = a\,\xi\,\eta,\ s = a\,[(\xi^2 - 1)\,(1 - \eta^2)]^{1/2}$$

$$\xi \geqslant 1.0,\ |\,\eta\,| \leqslant 1.0. \quad \ldots(11)$$

Here a is a constant. This transformation changes a circle in $\bar{w}$-plane to a front which becomes star shaped in $\bar{z}$-plane, as the wave approaches the centre. The curves ξ = constant are ellipses and η = constant are hyperbolas in the $\bar{w}$-plane. Transformation (10) changes circle to a star with $2n$ corners. We take a special case with $n = 2$, which makes the problem more handy.

Area A in two-dimentional space is

$$A = h_2 h_3 \quad \ldots(12)$$

where h_2, h_3 are metrical coefficients. This gives (Morse and Feshbach)

$$\frac{A}{A_0} = \left[\frac{\xi_0^2 + \eta^2 - 1}{\xi^2 + \eta^2 - 1}\right]^{1/4} \left[\frac{\xi^2 - \eta^2}{\xi_0^2 - \eta^2}\right]^{1/2} \quad \ldots(13)$$

Equations (8) and (13) combined give a relation between M_2, ξ and η. Here curves ξ = constant are ellipses and η = constant are hyperbolas along the direction of flow. We have computed equation (8) and (13) combined, for different values of η and ξ, and the results are plotted in the Figure 1. Value of the constant r is equal to 3 in all the calculations.

(§4) Discussions

Figure 1 shows that as ξ decreases, i.e., the wave front moves towards the centre, pressure ratio z increases. ξ kept constant, z also increases with η. For $\eta = 1$, pressure becomes too high. Point $\xi/\xi_0 = 0.1$ is the point at the centre. Thus the pressure at the centre increases with η. This is due to the non-symmetry of the shock front near the centre.

Conger (1968) has reported that there is not much increases in the wave velocity. But our calculations show the reverse, i.e., there is a definite increase in

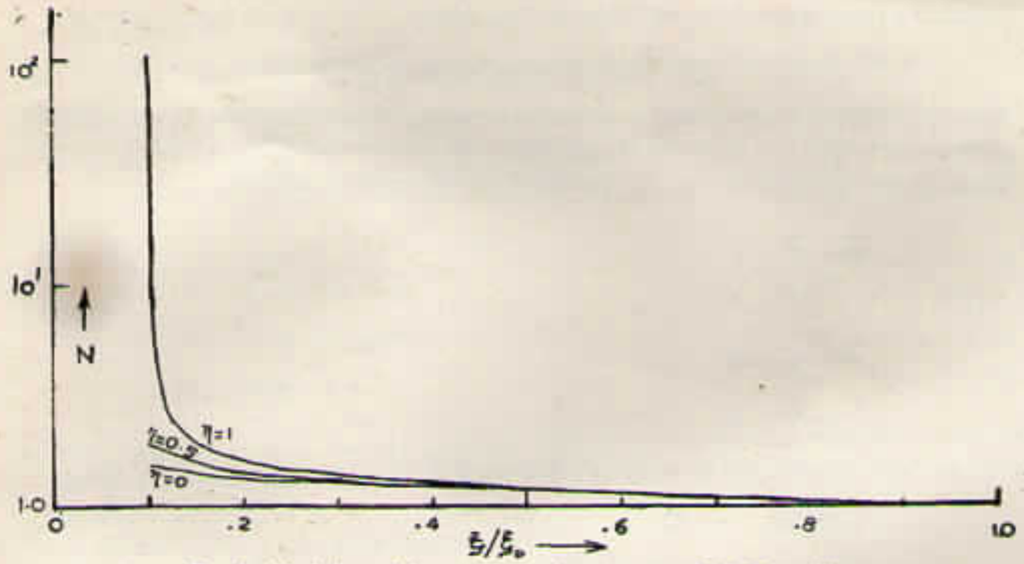

FIG. 1. Variation of the pressure ratio $p/p_0 = z$ *vs.* ξ/ξ_0 ($\xi_0 = 10$).

the wave velocity. The reason for different results is because we have not used the Chapman-Jouguet condition in our calculations.

ACKNOWLEDGEMENT

The author is thankful to Shri A. K. Madan for his interest, and to Shri P. K. Misra and M. S. Bola for their encouragement during the preparation of this paper. Thanks are also due to Director, TBRL, for providing facilities for carrying out this research work and also for giving permission to publish it.

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