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UNSTEADY FLOW BEHIND WEAK SHOCK WAVES IN WATER

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Abstract

Unsteady flow of water behind weak shock waves produced by the detonation of a spherical charge, is studied by the method of perturbation. Results are compared with those obtained by similarity methods elsewhere. Effect of gas bubble on the shock front is neglected in the present paper.

1. Introduction

The knowledge of flow profile behind the primary shock waves in water is of immense importance and also of academic interest. Using the method of similarity, Kochina and Melnikova [1, 2] have studied flow profile behind the shock waves, produced by the explosion and by the piston motion respectively. Assuming energy between the shock front and the piston surface to be variable, flow profile behind the shock in water has also been studied [4].

In the present paper, we have studied the unsteady motion of water behind the weak shocks, using perturbation method. Law of attenuation of shock was studied earlier in a series of papers by the first author [5, 6, 7], theoretically as well as experimentally. In the above paper and also in the present paper, effects of gas bubble on the flow are ignored.

Equations of motion of water are integrated by the method of perturbations. Two differential equations in the non-dimensional fluid parameters $f(\lambda, \Delta\rho)$, $g(\lambda, \Delta\rho)$ and λ are obtained, where f, g, and λ are non dimensional fluid velocity, density and distance, respectively. Expressing parameters f and g in terms of converging series, variation of those parameters with respect to λ and shock strength $\Delta\rho$ is obtained. Variation of parameters f and g behind spherical shock waves is shown in figures 1 and 2 for $\Delta\rho = 0.001$, 0.005 and 0.01 respectively, where $\Delta\rho$ is the jump in water density across the shock front. It is seen from figure 1 and 2 that as the distance ratio λ decreases 1 to 0.1, particle velocity ratio f increases continuously from its initial value but the density ratio g first increases and then decreases. These results are similar to those obtained earlier [4] in piston problem. We have not gone beyond the distance $\lambda = 0.1$ as the flow becomes divergent there. Moreover, there is a gas bubble of finite radius, at the centre, which we have ignored in the present paper.

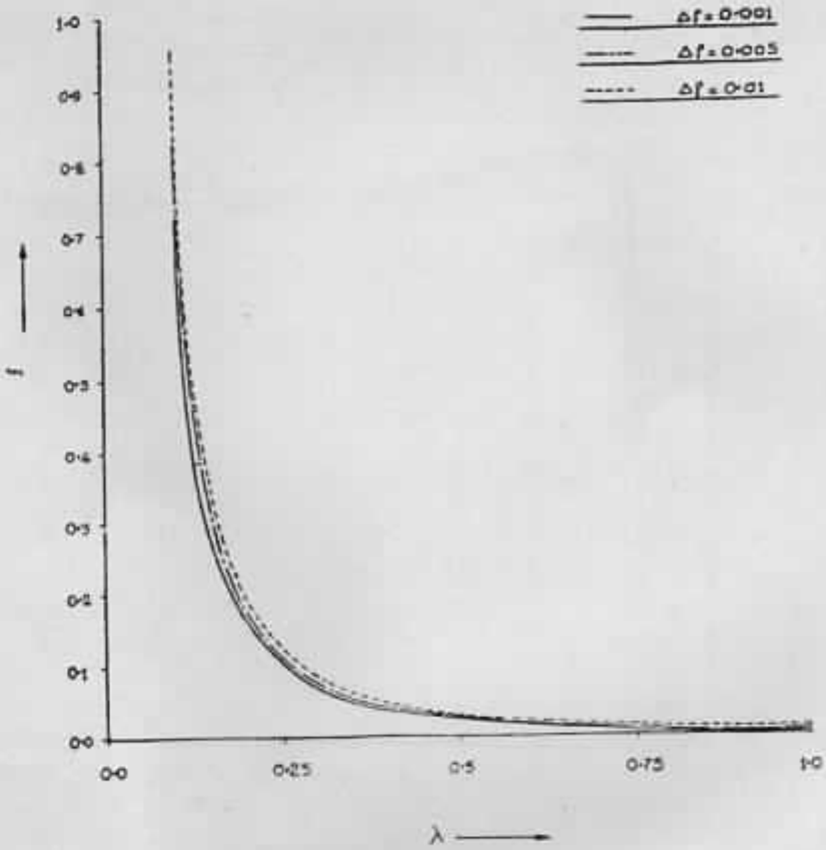

Fig. 1. Variation of parameter f versus λ

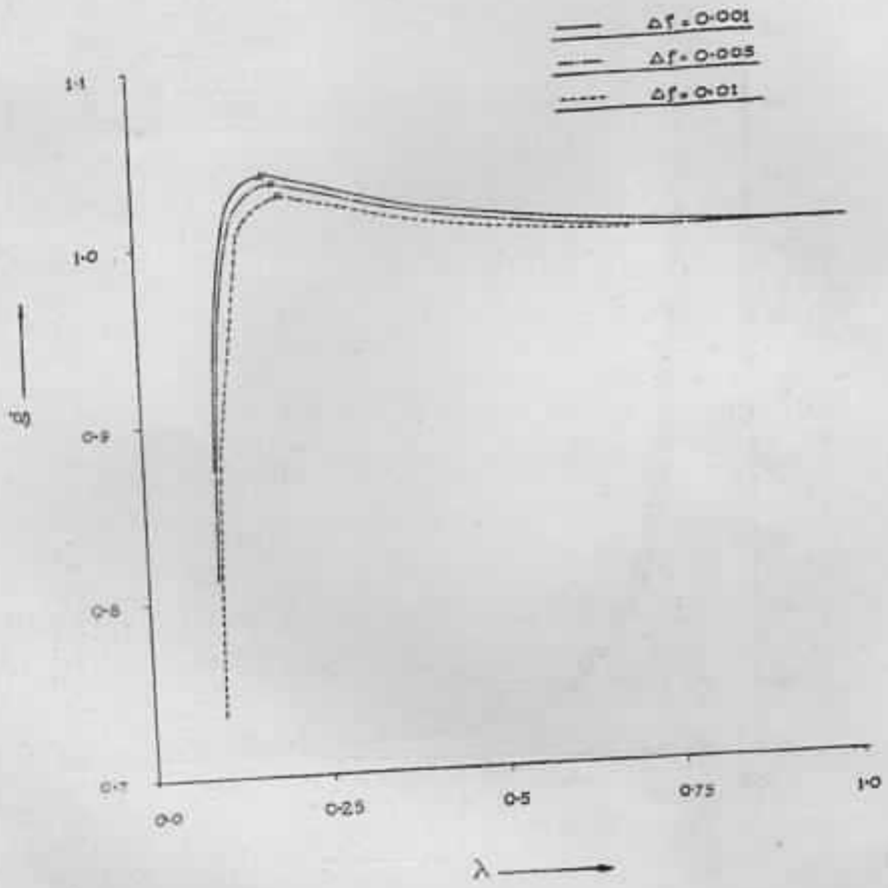

Fig. 2. Variation of parameter g versus λ

2. Basic Formulation of the Problem

We have earlier studied [7] the propogation and attenuation of spherical shock waves, produced by the detonation of an explosive charge in water. It is our aim in the present paper, to study the unsteady fluid motion behind the spherical shock waves. Basic equations governing this motion are,

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial r}(\rho u)+\frac{2\rho u}{r}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}=0 \tag{2}$$

$$\rho\frac{dE}{dt}=\frac{p}{\rho}\frac{d\rho}{dt} \tag{3}$$

Hugoniot equation of state of water is

$$U=a+bu_2 \tag{4}$$

where $a = 1.5565$, $b = 1.9107$ [3] and other symbols have usual meaning.

If at any time t, R is the radius of the shock front, then the jump conditions across the shock front are [7],

$$p_2=\rho_1 a^2\delta(\delta-1)/\{b-\delta(b-1)\}^2 \tag{5}$$

$$U=a\delta/\{b-\delta(b-1)\} \tag{6}$$

$$E_2^*=E^*_1+[(\delta-1)a/\{b-\delta(b-1)\}]^2 \tag{7}$$

where subscript 2 denotes values of fluid parameters behind the shock front and $\delta=\rho_2/\rho_1$ is the shock compression. Variation of shock compression δ with the shock radius is given by [7],

$$\delta[a(\delta-1)/\{b-\delta(b-1)\}]^2=3\alpha\bar{Q}J/(4\pi\rho_1\bar{R}^3) \tag{8}$$

where $\bar{Q}$ is the heat of explosion per unit volume, $\bar{R}=R/R_0$, R_0 being the radius of the undetonated explosive charge. Our aim in the present paper is to find the solution of equations of motions (1)–(4) with the help of boundary conditions (5)–(7).

We define a parameter $\Delta\rho$ as

$$\Delta\rho=(\rho_2-\rho_1)/\rho_1=\delta-1 \tag{9}$$

where $\Delta\rho$ is small for weak shocks. Using expression (9) in (8) we get after differentiation and expansion,

$$\frac{\partial\Delta\rho}{\partial R}=-\beta\Delta\rho/R \tag{10}$$

$$\beta = \beta_0 + \beta_1 \Delta\rho + \beta_2 \Delta\rho^2 + \ldots\ldots$$

where

$$\beta_0 = 3/2,\ \beta_1 = -3(2b-1)/4,\ \beta_2 = 3(4b-1)/8 \tag{11}$$

3. Discussion of the Problem

To find fluid parameters behind the shock front, we define non-dimensional parameters f, g and λ given as,

$$u/U = f(\lambda, \Delta\rho),\ \rho/\rho_2 = g(\lambda, \Delta\rho),\ r/R = \lambda \tag{12}$$

where f and g are functions of λ and $\Delta\rho$, r being the radial distance measured from the point of explosion. Substituting the parameters (12) in equations (1) and (2), one gets after some simplificatton

$$gf_\lambda + (f-\lambda)\, g_\lambda - \beta\Delta\rho g_{\Delta\rho} - g\beta\Delta\rho/(1+\Delta\rho) + 2fg/\lambda = 0 \tag{13}$$

$$\left(\frac{f}{U}\frac{\partial U}{\partial \Delta\rho} + f_{\Delta\rho}\right)\beta\Delta\rho - (f-\lambda)\, f_\lambda - \left\{\frac{\delta + b(\delta-1)}{\delta - b(\delta-1)}\right\}\frac{g_\lambda}{\delta^2 g} = 0 \tag{14}$$

where f_λ, g_λ, $f_{\Delta\rho}$, $g_{\Delta\rho}$ are partial differentials of f and g with respect to λ and $\Delta\rho$ respectively. Since f and g are functions of λ and $\Delta\rho$, we can write f and g in the form of converging series.

$$f(\lambda, \Delta\rho) = f_0(\lambda) + f_1(\lambda)\,\Delta\rho + f_2(\lambda)\,\Delta\rho^2 + \ldots\ldots\ldots \tag{15}$$

$$g(\rho, \Delta\rho) = g_0(\lambda) + g_1(\lambda)\,\Delta\rho + g_2(\lambda)\,\Delta\rho^2 + \ldots\ldots\ldots \tag{16}$$

where f_0, g_0, f_1, g_1 etc are functions of λ only.

After substituting the expressions (15) and (16) for f and g in (13) and (14) and comparing the coefficients of same powers of $\Delta\rho$, one gets coefficients of zeroth power of $\Delta\rho$ as,

$$g_0 f_0' + (f_0 - \lambda)\, g_0' + 2f_0 g_0/\lambda = 0 \tag{17}$$

$$(f_0 - \lambda)\, f_0' + g_0'/g_0 = 0 \tag{18}$$

and coefficients of $\Delta\rho$ as

$$g_0 f_1' + g_1 f_0' + f_1 g_0' + (f_0 - \lambda)\, g_1' - \beta_0\,(g_0 + g_1) + 2\,(f_1 g_0 + f_0 g_1)/\lambda = 0 \tag{19}$$

$$(f_0 - \lambda)\, f_1' + f_1 f_0' + (g_0'/g_0)(2b - 2 - g_1/g_0) + g_1'/g_0 - \beta\,(f_1 + bf_0) = 0 \tag{20}$$

Solving equations (17)–(20) for f_0', g_0' and f_1' g_1' one gets,

$$\frac{df_0}{d\lambda} = 2f_0/[\lambda\{(f_0 - \lambda)^2 - 1\}] \tag{21}$$

$$\frac{dg_0}{d\lambda} = -2f_0(f_0 - \lambda)\, g_0/[\lambda\{(f_0 - \lambda)^2 - 1\}] \qquad (22)$$

$$\frac{df_1}{d\lambda} = -[h_1(f_0 - \lambda) + h_2]/[g_0\{(f_0 - \lambda)^2 - 1\}] \qquad (23)$$

$$\frac{dg_1}{d\lambda} = [h_1 + (f_0 - \lambda)\, h_2]/[(f_0 - \lambda)^2 - 1] \qquad (24)$$

where

$$h_1 = f_1 g_0 f_0' + (2b - 2 - g_1/g_0)\, g_0' - \beta_0 g_0 (f_1 + b f_0) \qquad (25)$$

$$h_2 = -g_1 f_0' - f_1 g_0' + \beta_0 (g_0 + g_1) - 2(f_1 g_0 + f_0 g_1)/\lambda \qquad (26)$$

Equations (21)-(24) are four simultaneous differential equatious in f_0, g_0, f_1 and g_1. Boundary conditions for f_0, g_0 and f_1, g_1 are obtained from relations (5)-(6), i.e. the jump conditions across the shock front. At the shock front where $\lambda = 1$, we have

$$f_0 = 0 \qquad f_1 = 1$$

$$g_0 = 1 \qquad g_1 = 0 \qquad (27)$$

We have integrated the differential equations (21)-(24) subject to the boundary conditions (27), numerically and the results are shown in the Figures 1 and 2.

4. Discussion and the conclusions

In figures 1 and 2, we have shown the variation of the parameters f and g versus λ for $\Delta\rho = 0.001$, 0.005 and 0.01 respectively. Corresponding pressures behind the shock front in water are 24.00, 123.124 and 249.753 atmospheres. It is seen from figure 1 that particle velocity ratios $u/U = f(\lambda)$ increases continuously as λ decreases from 1 to 0.1. In figure 2, variation of density ratio $g(\lambda)$ is shown. It is seen that for $\Delta\rho = 0{,}001$, g first increases and when $\lambda = 0.19$, it starts decreasing. This behaviour of the density variations is similar to that of pioston problem [4]. As the strength of the shock increases, this bend of the parameter $g(\lambda)$, shifts away from the shock front i.e. towards the centre.

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