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FLOW BEHIND WEAK AND STRONG SHOCK WAVES IN WATER

V. P. SINGH AND A. M. N. YOGI

Centre for Aeronautical Systems Studies and Analyses, C V Raman Nagar Bangalore 560075

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Unsteady flow water behind weak and strong shock waves produced by the detonation of a spherical charge, is studied by the method of perturbations. Results are compared with those obtained by similarity methods elsewhere. Effect of gas bubble on the shock front is neglected in the present paper.

1. INTRODUCTION

The knowledge of flow profile behind the primary stock waves in water is of immense importance and also of academic interest. Using the method of similarity, Kochina and Melnikova[1,2] have studied flow profile behind the shock waves, produced by the explosion and by the piston motion respectively, and by perturbation method, this problem is solved for flow behind shocks in air by Singh[5]. Assuming energy between the shock front and the piston surface to be variable, flow profile behind the shock in water has also been studied[4].

In the present paper, we have studied the unsteady motion of water behind the shock waves, using perturbation method. Law of attenuation of shock was studied earlier in a series of papers by the first author[6 8], theoretically as well as experimentally. In the above papers and also in the present paper, effects of gas bubble on the flow are ignored.

Flow behind shock waves is governed by the equations of motion of compressible fluids which are integrated by the methods of perturbations, taking shock front as one of the boundary. Equations of motion are reduced to two differential equations in the non-dimensional fluid parameters $f(\lambda, \Delta\rho)$, $g(\lambda, \Delta\rho)$ and λ, where f, g, and λ are non-dimensional fluid velocity, density and distance respectively. Expressing parameters f and g in the form of converging series, variation of these parameters with respect to λ and shock strength $\Delta\rho$ is obtained. Variation of parameters f and g behind spherical shock waves is shown in Figures 1 to 4 for the two cases of weak and strong shocks respectively. These results are similar to those obtained earlier[4] in piston problem.

2. BASIC FORMULATION OF THE PROBLEM

We have earlier studied[8] the propagation and attenuation of spherical shock waves, produced by the detonation of an explosive charge in water. It is our aim in the present paper, to study the unsteady fluid motion behind the spherical shock waves. Basic equations governing this motion are,

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial r}(\rho u) + \frac{2\rho u}{r} = 0 \qquad \text{...(1)}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 \qquad \text{...(2)}$$

$$\rho \frac{dE}{dt} = \frac{p}{\rho}\frac{d\rho}{dt} \qquad \text{...(3)}$$

Hugoniot equation of state of water is

$$U = a + bu_2 \qquad \text{...(4)}$$

where $a = 1.5565$, $b = 1.9107$ (Madan[3]) and other symbols have usual meaning.

If at any time t, R is the radius of the shock front, the jump conditions across the shock front are[8],

$$p_2 = \rho_1 a^2 \delta(\delta - 1)/\{b - \delta(b-1)\}^2 \qquad \text{...(5)}$$

$$U = a\delta/\{b - \delta(b-1)\} \qquad \text{...(6)}$$

$$E_2^* = E_1^* + [(\delta - 1)a/\{b - \delta(b-1)\}]^2 \qquad \text{.. (7)}$$

where subscript 2 denotes values of fluid parameters behind the shock front and $\delta = \rho_2/\rho_1$ is the shock compression. Variation of shock compression δ with the shock radius is given by[8].

$$\delta[a(\delta - 1)/\{b - \delta(b-1)\}]^2 = 3\alpha\bar{Q}J/(4\pi\rho_1\bar{R}^3) \qquad \text{...(8)}$$

where $\bar{Q}$ is the heat of explosion per unit volume, $\bar{R} = R/R_0$, R_0 being the radius of undetonated explosive charge. Flow behind the shock wave is governed by equations of motion (1)—(4). Thus our aim in the present paper is to find the solution of these equations with the help of boundary conditions (5)—(7).

We define the shock strength by a parameter $\Delta\rho$ so that

$$\Delta\rho = (\rho_2 - \rho_1)/\rho_1 = \delta - 1 \qquad \text{.. (9)}$$

where $\Delta\rho$ is assumed to be very small for weak shocks and for strong shocks $0 < \Delta\rho < 1$. Largest value of $\Delta\rho$ is 0.7 for the available conventional explosives. Using expression (9) in (8) we get after differentiation and expansion (Appendix A).

$$\frac{\partial \Delta\rho}{\partial R} = -\beta\Delta\rho/R \qquad \text{.. (10)}$$

$$\beta = \beta_0 + \beta_1\Delta\rho + \beta_2\Delta\rho^2 + \ldots$$

where

$$\beta_0 = 3/2,\ \beta_1 = -\ 3\,(2b - 1)/4,\ \beta_2 = 3\,(4b - 1)/8. \qquad \text{...(11)}$$

3. Discussion of the Problem

To evaluate the variation of fluid parameters behind the shock front, we define non-dimensional parameters f, g and λ as,

$$u/U = f(\lambda, \Delta\rho),\ \rho/\rho_2 = g(\lambda, \Delta\rho),\ r/R = \lambda \qquad \text{...(12)}$$

where f and g are functions of λ and $\Delta\rho$, r the radial distance measured from the point of explosion and u,ρ the fluid velocity, density at distance are respectively. Substituting the parameters from (12) in equations (1) and (2), one gets after some simplification (Appendix A)

$$gf_\lambda + (f - \lambda)\, g_\lambda - \beta\Delta\rho\, g_{\Delta\rho} - g\beta\Delta\rho\,(1 + \Delta\rho)^{-1} + 2fg\,\lambda^{-1} = 0 \quad \text{...(13)}$$

$$\left(\frac{f}{U}\frac{\partial U}{\partial \Delta\rho} + f_{\Delta\rho}\right)\beta\Delta\rho - (f - \lambda)\, f_\lambda - \left\{\frac{\delta + b\,(\delta - 1)}{\delta - b\,(\delta - 1)}\right\}\frac{g_\lambda}{\delta^2 g} = 0 \qquad \text{...(14)}$$

where f_λ, g_λ, $f_{\Delta\rho}$, $g_{\Delta\rho}$ are partial derivatives of f and g with respect to λ and Δ respectively. Since f and g are functions of λ and $\Delta\rho$, we can write f and g in the form of converging series.

$$f(\lambda, \Delta\rho) = f_0(\lambda) + f_1(\lambda)\,\Delta\rho + f_2(\lambda)\,\Delta\rho^2 + \dots \qquad \text{...(15)}$$

$$g(\lambda, \Delta\rho) = g_0(\lambda) + g_1(\lambda)\,\Delta\rho + g_2(\lambda)\,\Delta\rho^2 + \dots \qquad \text{...(16)}$$

where f_0, g_0, f_1, g_1 etc are functions of λ only.

After substituting the expressions (15) and (16) for f and g in (13) and (14) and comparing the coefficients of same powers of $\Delta\rho$, one gets equations of different order as zeroth order equations.

$$g_0 f_0' + (f_0 - \lambda)\, g_0' + 2f_0 g_0 \lambda^{-1} = 0 \qquad \text{...(17)}$$

$$(f_0 - \lambda)\, f_0' + g_0' g_0^{-1} = 0. \qquad \text{...(18)}$$

First order equations

$$g_0 f_1' + (f_0 - \lambda)\, g_1' - h_2 = 0 \qquad \text{...(19)}$$

$$g_0 (f_0 - \lambda)\, f_1' + g_1' + h_1 = 0. \qquad \text{...(20)}$$

Second order equations

$$g_0 f_2' + (f_0 - \lambda)\, g_2' - h_4 = 0 \qquad \text{...(21)}$$

$$g_0 (f_0 - \lambda) f_2' + g_2' + h_3 = 0 \qquad ...(22)$$

where

$$h_1 = f_1 g_0 f_0' + (2b - 2 - g_1 g_0^{-1}) g_0' - \beta_0 g_0 (f_1 + b f_0)$$

$$h_2 = - g_1 f_0' - f_1 g_0' + \beta_0 (g_0 + g_1) - 2 (f_1 g_0 + f_0 g_1)/\lambda$$

$$\begin{aligned} h_3 = {} & f_2 g_0 f_0' + f_1 g_0 f_1' \\ & + g_0' [g_1^2 g_0^{-2} - g_2 g_0^{-1} - g_1 (2b - 2) g_0^{-1} + 2b^2 - 6b \\ & + 3] - g_1' (g_1 g_0^{-1} - 2b + 2) \\ & - \beta_0 g_0 [f_0 (b^2 - 2b) + f_1 b + 2 f_2] \\ & - \beta_1 g_0 (b f_0 + f_1) \end{aligned} \qquad ...(23)$$

$$\begin{aligned} h_4 = {} & - g_2 f_0' - g_1 f_1' - f_2 g_0' - f_1 g_1' \\ & - \beta_0 (g_0 - g_1 - 2g_2) + \beta_1 (g_0 + g_1) \\ & - 2 (f_0 g_2 + f_1 g_1 + f_2 g_0) \lambda^{-1}. \end{aligned}$$

Solving eqns. (17)—(22) for f_0, g_0, f_1 g_1 and f_2, g_2 one gets

$$\frac{df_0}{d\lambda} = 2 f_0/[\lambda \{(f_0 - \lambda)^2 - 1\}] \qquad ...(24)$$

$$\frac{dg_0}{d\lambda} = - 2 f_0 (f_0 - \lambda) g_0/[\lambda \{(f_0 - \lambda)^2 - 1\}] \qquad ...(25)$$

$$\frac{df_1}{d\lambda} = - [h_1 (f_0 - \lambda) + h_2] /[g_0 \{(f_0 - \lambda)^2 - 1\}] \qquad ..(26)$$

$$\frac{dg_1}{d\lambda} = [h_1 + (f_0 - \lambda) h_2]/[(f_0 - \lambda)^2 - 1] \qquad ...(27)$$

$$\frac{df_2}{d\lambda} = - [h_3 (f_0 - \lambda) + h_4]/[g_0 \{(f_0 - \lambda)^2 - 1\}] \qquad ...(28)$$

$$\frac{dg_2}{d\lambda} = [h_3 + h_4 (f_0 - \lambda)]/[(f_0 - \lambda)^2 - 1]. \qquad ...(29)$$

Equations (24)—(29) are six simultaneous differential equations in f_0, g_0, f_1, g_1, f_2 and g_2. Boundary conditions for f_0, g_0, f_1, g_1, f_2 and g_2 are obtained from relations (5)—

(6), i. e. the jump conditions across the shock front. At the shock where $\lambda = 1$, we have

$$f_0 = 0 \qquad f_1 = 1 \qquad f_2 = -1$$

$$g_0 = 1 \qquad g_1 = 0 \qquad g_2 = 0. \qquad \ldots(30)$$

We have integrated the differential equations (24) — (29) subject to the boundary conditions (30), using Runge-Kutta method of fourth order the results are shown in the Figs. 1 to 4.

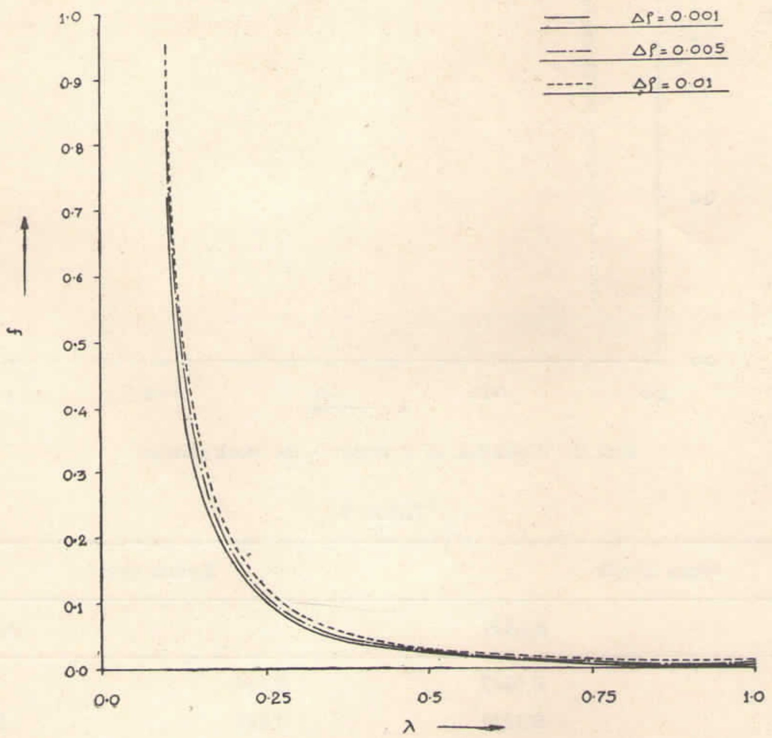

FIG. 1. Variation of f versus λ for weak shocks.

4. DISCUSSION AND THE CONCLUSIONS

In Figs. 1 and 2, we have shown the variation of the parameters f and g versus λ for $\Delta\rho = 0.001$, 0.005 and 0.01 respectively. This is the case of weak shocks. In Table I, values of shock pressure p_2 are shown for various values of $\Delta\rho$.

It is seen from Fig. 1 that particle velocity ratios $u/U = f(\lambda)$ increases continuously as λ decreases from 1 to 0.1, value of f being approximately zero at $\lambda = 1$ for weak shocks. In Fig. 2, variation of density ratio $g(\lambda)$ is shown. It is seen that for $\Delta\rho = 0.001$, g first increases and when $\lambda = 0.19$, it starts decreasing. The trend of the

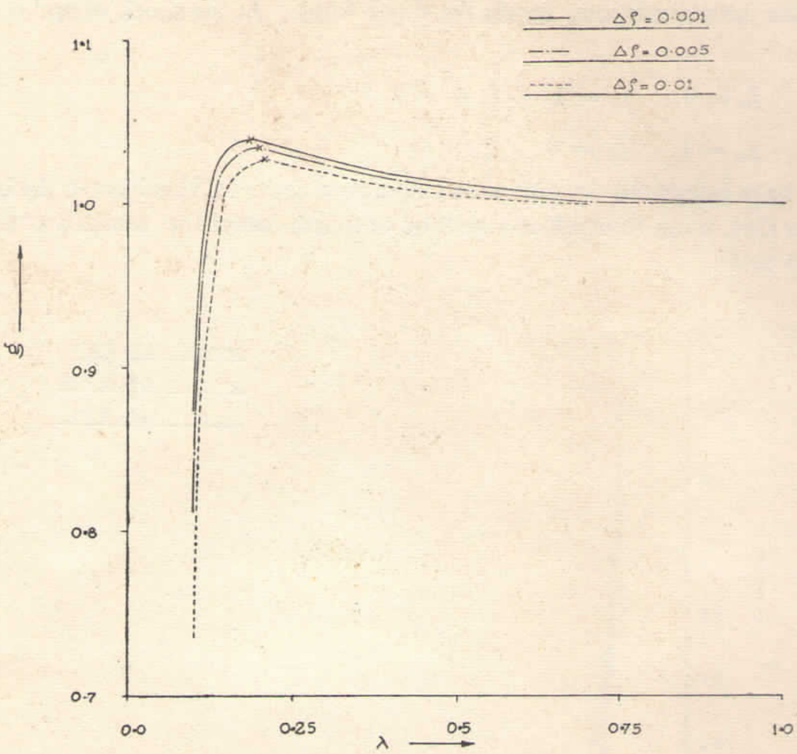

FIG. 2. Variation of g versus λ for weak shocks.

TABLE I

Weak Shock		Strong shock	
$\Delta\rho$	$p_2(Kb)$	$\Delta\rho$	$p_2(Kb)$
0.001	0.0243	0.150	5.6062
0.005	0.1229	0.250	12.6925
0.010	0.2492	0.350	24.6650

density variations in the present problem is similar to that of piston problem[4]. In Fig. 3 f is plotted versus λ for the case of strong shock waves. For $\lambda = 1$, value of $f = u_2/U = (\delta - 1)/\delta$, which first decreases, then starts increasing exponentially, as λ decreases from 1 to zero. Similar trend is found in the variation of g for strong shocks.

Once conditions at the shock front are known, variation of fluid parameters is known from shock front to the point of explosion. Conditions at the shock front are functions of shock radius, which is given by Singh *et al.*[8].

In the present work we have ignored the presence of gas bubble at the centre.

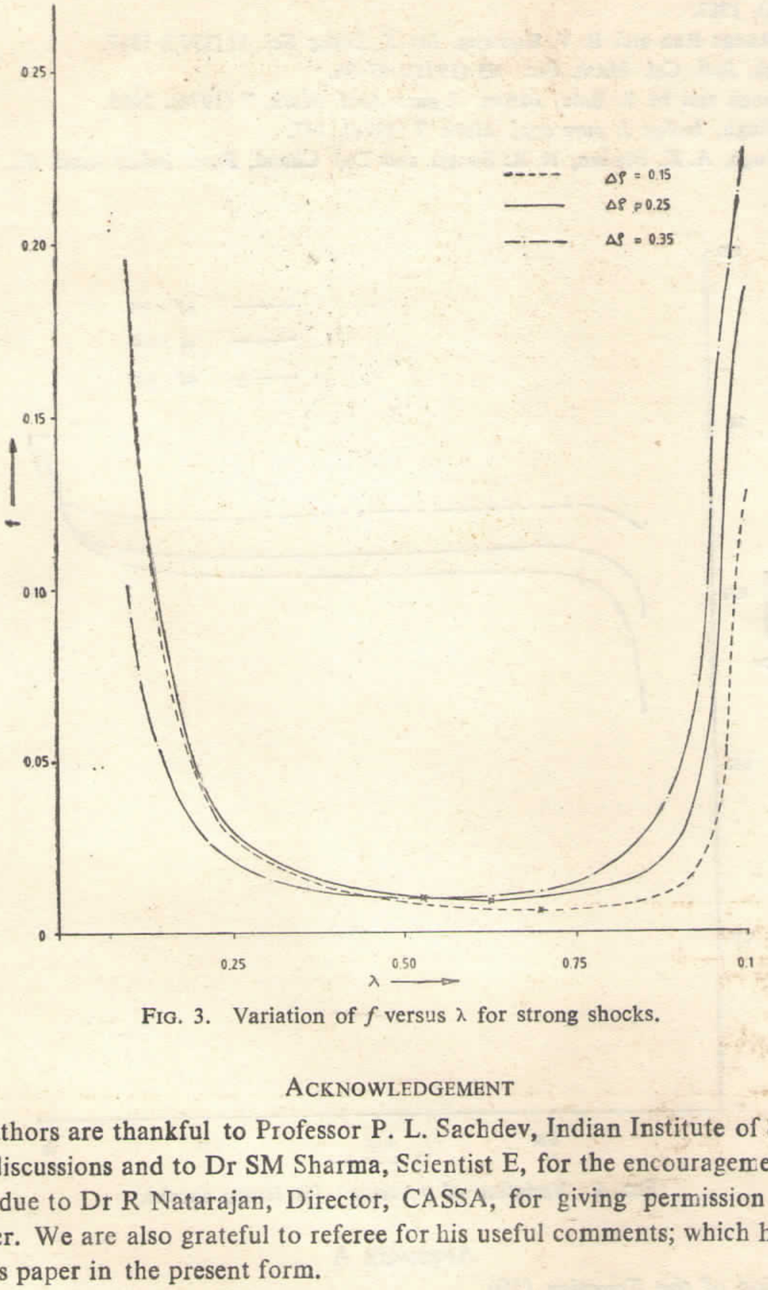

FIG. 3. Variation of f versus λ for strong shocks.

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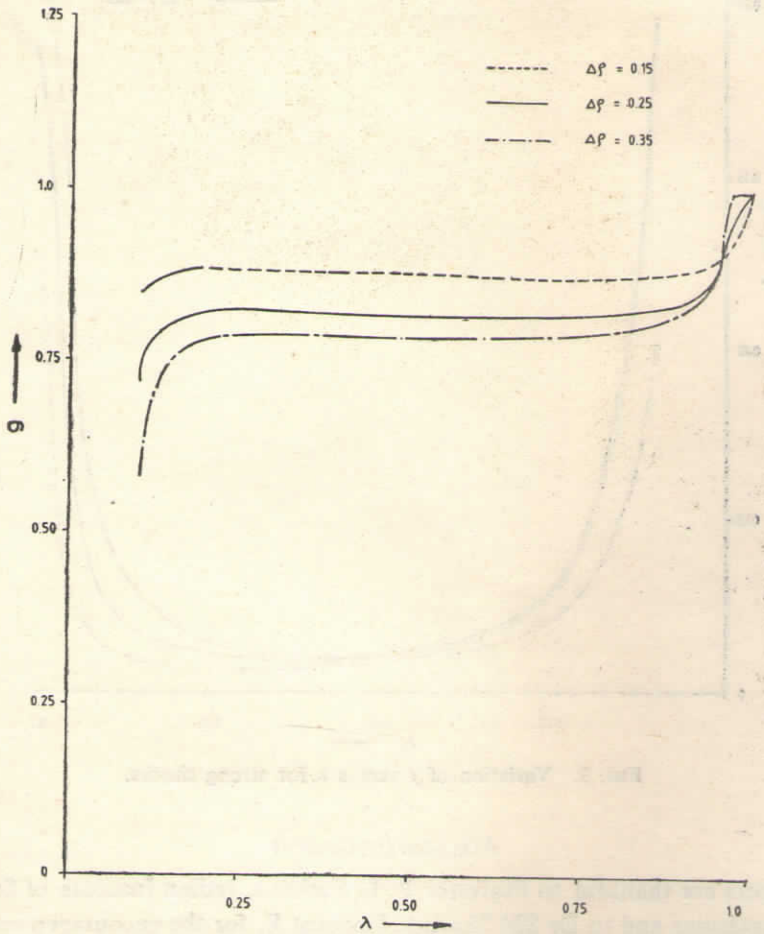

FIG. 4. Variation of g versus λ for strong shocks.

APPENDIX A

1. *Derivation of the Equation* (10)

Substituting equation (9) in (8) we get

$$\frac{(1 + \Delta\rho)\,\Delta\rho^2}{[1 + \Delta\,(1 - b)]^2} = \frac{K}{R^3} \qquad \ldots(A.1)$$

where

$$K = 3\alpha \bar{Q} J R_0^3 / 4\pi \rho_1 a^2. \qquad \text{...(A.2)}$$

Taking logs and differentiating (A.1) with respect to R we get

$$\frac{\partial \Delta\rho}{\partial R} = - \frac{3\Delta\rho}{2R} [1 + \Delta\rho (2 - b) + \Delta\rho^2 (1 - b)] [1 + \tfrac{3}{2} \Delta\rho + \frac{(1 - b)}{2} \Delta\rho^2]. \qquad \text{...(A.3)}$$

Expanding the right handside and rearranging the coefficient of $\Delta\rho$ and $\Delta\rho^2$ etc. we get

$$\frac{\partial \Delta\rho}{\partial R} = - \frac{\beta\Delta\rho}{R}$$

Where

$$\beta = \beta_0 + \beta_1 \Delta\rho + \beta_2 \Delta\rho^2 + \ldots$$

$$\beta_0 = 3/2$$

$$\beta_1 = -3 (2b - 1)/4$$

$$\beta_2 = 3 (4b - 1)/8.$$

2. *Derivation of Equation* (13) *and* (14)

Since entropy variations are negligible in underwater shocks, we have

$$\frac{\partial p}{\partial r} = \left(\frac{\partial p}{\partial \rho} \right)_s \frac{\partial \rho}{\partial r} = c^2 \frac{\partial \rho}{\partial r}$$

where

$$c^2 = \partial p / \partial \rho)_s$$

is the sound velocity in compressed water.

Using equation (5) we get

$$c^2 = \frac{\partial p_2}{\partial \rho_2} = \frac{a^2 [\delta + b (\delta - 1)]}{[\delta - b (\delta - 1)]^3}.$$

In equations (1) and (2) independent parameters r and t are transformed to non-dimensional parameter λ and $\Delta\rho$ by the following operators

$$\frac{\partial}{\partial t} = - \frac{U\lambda}{R} \frac{\partial}{\partial \lambda} - \frac{\beta U}{R} \Delta\rho \frac{\partial}{\partial \Delta\rho}$$

$$\frac{\partial}{\partial r} = \frac{1}{R} \frac{\partial}{\partial \lambda}.$$

Using (A.5) alongwith (A.4), (9) (10) and (12) in equations (1) and (2) we get equation (13)—(14) after some simplifications.

3. *Comparision with Ref.* Ranga and Ramana[4]

Although problem in reference 4 is dealt using similarity methods, but trend in the variations of fluid parameters behind the shock in piston problem and explosion problem should be comparables. The similarity parameter λ in ref.[4] is same as λ of our paper, as follows. In reference[4] equation (33) and (18) are as

$$\lambda = (A/\rho_1)^{\delta/2}\, r t^{-\delta} \qquad \text{...(33)}$$

$$r_2 = \alpha t^{\delta}. \qquad \text{...(18)}$$

Eliminating t from (18) and (33) we get

$$\lambda = (A/\rho_1)^{\delta/2}\, \alpha\, (r/r_2)$$

Now at the shock front $r = r_2$, $\lambda = 1$

$$\therefore\ (A/\rho_1)^{\delta/2}\, \alpha = 1.$$