...from:
Anti Submarine Warfare
Feb 1985 - IAT PUNE

INTERACTION OF EXPLOSIVE SHOCKS WITH DEEP SEA TARGETS

VP Singh & DS Murty
Terminal Ballistics Research Laboratory
Chandigarh 160020

Pressure time history on the boundary of submarine or any under water structure, due to the detonation of an explosive charge in deep sea is of immense importance in Anti Submarine Warfare. In the present paper, we have tackled this problem in two parts. In part one, we have studied attenuation of shock wave from the point of explosion upto the target surface. In the second part, interaction of this shock with the boundary of target is studied. For simplicity, we have assumed that the ~~shape of the~~ target is cylindrical. It is also assumed that the shock front, which interacts with the cylinder is a plane front. Whitham's method of shock-shock propagation is used to study the reflected pressure around the cylindrical surface. It is found that if at an angle $\psi = 0$ interaction is normal, then from $\psi = 0$ to $\psi = \psi_{crit}$, reflection is regular, but ~~yegone~~ beyond ψ_{crit}, there is no regular reflection and Mach stem is formed. Velocity of this Mach stem and pressure on the surface is determined theoretically.

INTRODUCTION

1. Determination of pressure-space-time history around targets in deep sea, due to the explosive shock loading is of great interest in Naval research. Using Whitham's method of shock-shock propagation (Whitham 1957, 1959), we have found the pressure-space history around the circular cylinder in water, exerted by an explosive shock, due to an explosion in the vicinity of the cylinder. Whitham (1959), in his paper discussed a new technique, by which one can find the length and velocity of Mach stem of triple shock on the circular cylinder. This problem wqs later varified experimentally in air by Bryson et.al. (1961). In the above theory, only Mach stem length is found and it is assumed that for small

angles, Mach stem length λ is equal to $\frac{1}{2} \sin^{n+1} \psi$. But it is known that at $\psi = 0$ pressure on the cylinder is the reflected pressure (Figure 2). We in the present paper, have tried to evaluate the pressure on the surface of the cylinder. We have assumed that the shock is plane and it first meets a point on the cylinder in two dimensional space. At this point, pressure on the cylinder is the reflected pressure. The reflected pressure is evaluated as ψ increases. It is found that the angle ψ crit, beyond which there is no regular reflection, increases as the shock strength increases. Beyond ψ crit Mach stem is formed. Velocity and length of the Mach stem is determined by Whitham's method of shock-shock propagation, for various shock strength. Pressure on the cylinder is also evaluated for different shock strengths. It is found that the pressure of Mach stem first increases and after achieving maxi- mum, then comes down to incident shock pressure at ψ =90 degree.

ATTENUATION AND PROPAGATION OF SHOCK WAVES IN WATER

2. Attenuation and propagation of spherical shock waves in water, due to underwater explosion, is studied by various authors. Details of these works are given in the review papers by the present author (Singh 1981, 1982). We will discuss only one of the methods, which is due to Whitham (Whitham (1958).

3. Let us assume that there is an explosion in water, due to which a spherical shock is produced. The jump conditions across this shock at any time t are,

$$p_2 = \frac{\rho_1 U (U - a)}{b} \qquad 1.1$$

$$u_2 = \frac{U - a}{b} \qquad 1.2$$

$$\rho_2 = \frac{\rho_1 \, b \, U}{\{a + (b-1) U\}} \qquad 1.3$$

where P_2, ρ_2, u_2, U are pressure, density, particle velocity, and shock velocity across the shock front and a, b are the experimental parameters of Hugoniot form of equation of state (Singh et al 1980). In the present paper, values of a and b are taken as $a = 1.556$ mm/μs and $b = 1.9106$. To find the relation between the shock radius R and the Mach number of the shock front velocity i.e. M ($M=U/a$), we substitute equation (1.1)-(1.3) in the equation of motion along the positive characteristic axis, i.e.

$$dp_2 + P_2 C_2 du_2 + \frac{P_2 C_2^2 u_2}{(u_2 + C_2)} \frac{dA}{A} = 0 \qquad 1.4$$

where c_2 is sound velocity and $A = 4\pi R^2$, R being the radius of shock front at time t. Equation (1.4) combined with (1.1)-(1.3) gives after some simplifications,

$$-A \frac{dM}{dA} = MK(M) \qquad (1.5)$$

where

$$K(M) = \frac{[1 + (b-1) M]}{\left[1 + \left\{\frac{1+(b-1)M\}\sqrt{2M-1}}{(M-1)}\right]\left[1+\frac{M}{\sqrt{2M-1}}\right]} \qquad (1.6)$$

Integrating (1.5) one gets

$$A/A_0 = f(M)/f(M_0) \qquad (1.7)$$

where

$$f(M) = \exp\left\{-\int \frac{dM}{MK(M)}\right\}$$

It is seen that function K(M) varies from 0 to 1 as M varies from 1 to infinity. Relation (1.7) is a relation between the Mach number and the area change. It is shown by Singh (1982) that attenuation in water by Whitham's method does not conform with the experimental data, but due to simplicity of the method, we are using this method in our paper. Attenuation can also be found by scaling laws (Murty et. al 1976), or Energy Hypothesis (Singh 1976), if they are expressed in the form given by the equation (1.7).

REGULAR REFLECTION OF SHOCK WAVES FROM A CIRCULAR BOUNDARY

4. In this section, we first derive relations for the reflection of plane shock front in water from a plane boundary . These results will be applied to the reflection of shock front from the surface of cylindrical boundaries. Let IO be the incident plane shock in figure 1, meeting the wall at point O at an angle ψ . After reflection this shock is shown as OR, which moves with the velocity U_2. Velocity of point O is given as

...t O should
...e stationary
along the wall
for reflection.
...ocity of
...flected
...ock ||
wall is
...ual to
...at of incident
shock.

$$\{U_2 + u_2 \cos(\pi - \rho - \alpha)\} \operatorname{cosec} \alpha = U \operatorname{cosec} \psi \quad (2.1)$$

where u_2 and u_3 are the particle velocities behind the incident and the reflected shocks respectively. Since the fluid flow must be parallel to the wall, we have

$$u_2 \cos \psi = u_3 \cos \alpha \quad (2.2)$$

From relations (1.1)-(1.3), (2.1),(2.2) one gets after some algebraic calculations,

$$\begin{aligned}(b-1)(M-1)\sin\psi\cos\psi + \sin\alpha\cos\alpha\Big[\frac{(M-1)}{b}\sin^2\psi \\ -M + (M-1)\sin\psi\cos\psi\tan\alpha\Big] = 0\end{aligned} \quad (2.3)$$

Equation (2.3) gives a relation between the incident angle ψ and the reflected angle α for a given M. Knowing α reflected pressure can be obtained from the relations (1.1), (1.2) and (2.2)

5. We assume that a small section of a circular cylinder acts as a plane surface approximately and shock front is reflected from

from this part as discussed above.

6. Let 0 be the centre of the cross section of the circular cylinder of radius unity (Figure 2). Let at time t=0, a plane shock front hits the cylinder at an angle $\psi = 0$. At this point shock front will have normal reflection. At any angle ψ, we assume that a plane shock hits the small segment of the cylinder. Thus the angle between the shock front and the segment of the cylinder is ψ, which is the incident angle of the shock front (Figure 2).

7. Equation (2.3) gives reflected angle α for the given incident angle ψ and shock strength M. Once α is known, u_2 can be found from the relations (1.1)-(1.3) along with (2.2). It is seen that as ψ increases from 0 onward,, there is regular reflection but after some criticle value of ψ, there is a Mach stem formation. This angle we denote by ψ_{crit}.

MACH REFLECTION FROM THE CYLINDER'S SURFACE

8. In the present section we will develop a theory for the propagation of Mach stem along the cylindrical surface as ψ increases from 0 to $\frac{\pi}{2}$. Whitham (1959) has devised a method for finding the length of the Mach stem of the triple shock around a circular cylinder in air. In this section, we modify this theory for the case of water and find the pressure aound the cylinder immersed in water.

9. If M is the Mach number of the Mach stem whose length is λ at an angle ψ . We have

$$\frac{d\lambda}{d\psi} = (1+\lambda)\tan\psi - \left(1+\frac{\lambda}{2}\right)\frac{M}{M_\lambda}\sec\psi \qquad (3.1)$$

$$\frac{1}{\lambda} = \left[\frac{f(M)}{f(M_\lambda)}\operatorname{cosec}\psi - 1\right] \qquad (3.2)$$

where f(M) is given by the relation (1.8). Differentiating (3.2) and using (1.8) & 3.1, one gwts after some simplifications

$$\frac{d M_\lambda}{d\psi} = \frac{-M K(M)}{\sin\psi}\left[\left\{\frac{f(M)}{f(M_\lambda)} - \sin\psi\right\}\left\{\tan\psi - \frac{(2+\lambda)}{2(1+\lambda)}\sec\psi\, M/M_\lambda\right\} - \cos\psi\right] \qquad (3.3)$$

Equations (3.2) and (3.3) can be integrated simultaneously to give variation of M_λ and λ versus ψ. Variation of λ versus ψ is shown in figure 3 for M = 4. We have integrated these equations from ψ_{crit} to ψ =90 degree. Initial values are taken to be $\lambda = 0$, $\psi = \psi_{crit}$ and thus M_λ is obtained from (3.2). Variation of pressure for regular reflection and Mach reflection are shown in figure 4.

RESULTS AND DISCUSSIONS

10. Bryson (1961) assumed in his work that Mach stem is formed right at $\psi = 0$. But this point is a singular point (equation (3.2) and pressure is infinite at this point. In actual practice there is normal reflection at $\psi = 0$. We have found the reflected pressure at the cylinder as ψ increases from zero to ψ_{crit}. Beyond this point, equation (2.3) has no solution. This shows that Mach stem is formed only at point $\psi = \psi_{crit}$. In the table 1 we have shown ψ_{crit} versus Mach number M.

TABLE 1

M	ψ_{crit} Degrees	α_{crit} Degrees	Pressure on the cylinder at (kilobars)
3	39	33	103.0139
4	32	33.65	258.7756
5	30	31.42	447.7183

11. It is seen from figure 3, that initially increase in is very slow but becomes very large as ψ spproaches 90 degrees.

12. Kinney (1962) has stated that pressure behind the Mach stem is 20% less than that of pressure behind the reflected shock. Write (1961) has also given experimental curves for the variation of Mach stem pressure in air. We have assumed in the present paper that the Mach stem pressure is a linear combination of pressures behind incident and the reflected shock. Variation of regular reflected pressure and Mach stem pressure is shown in girure 4 for M=3, 4 and 5. It is seen that the Mach stem pressure first increases and attains a maximum and then decreases upto the incident pressure at $\psi = \frac{\pi}{2}$. It is an interesting observation that maximum pressure is at ψ =60-65. However this need experimental varifications. Trends of our curve agree with that given by Write (1961).

CONCLUSION

13. Problem of finding the pressure history on targets in sea, due to shocks is new and challenging. Even theory for the evaluation of the Mach stem pressure is not fully developed so far. We too have made several assumptions in the present problem for the evaluation of pressure around the cylinder. We have assumed that shock front is plane and obeys the laws of reflection of plane shock from a plane boundary. These assumptions are not exactly true in actual practice.

ACKNOWLEDGEMENT

14. We are thankful to Col. Harinder Singh for his interest and Sh. J.P. Sirpal, Director for giving permission to publish this work. Thanks are also due to Sh S.K. Shukla for his help in numerical calculations.

[RE]FERENCES

[1]. Bryson, AE and RWF Gross (1961) J.Fluid Mech.,10, p.1-16.
[2]. Kinney, GF (1962): Explosive shocks in air, Mc Millan Co., NY, p. 179.
[3]. Murty DS et.al (1976): TBRL Report No. 139/76 (Classified)
[4]. Singh, VP (1976): Ind. J. Pure Appl. Maths., 7, 147-50
[5]. Singh, VP (1980): Proc. Ind. Acad. Sci.(Eng.Sci), 3,pt 2 p. 169-75.
[6]. Singh, VP (1981): On under water explosion, in Workshop on Terminal Ballistics, IAT, Pune. p.103.
[7]. Singh, VP (1982): Def. Sci. J., 32, p. 327-32.
[8]. Whitham, GB (1955): J. Fluid Mech., 2, p. 145-71
[9]. Whitham, GB (1958): ibid, 4, p 337.
[1]0. Whitham, GB(1969) : ibid, 5, 369-80
[1]1. Write, JK (1961) : Shock Tubes, Mathew and Co., NY, 72-73

-.-.-

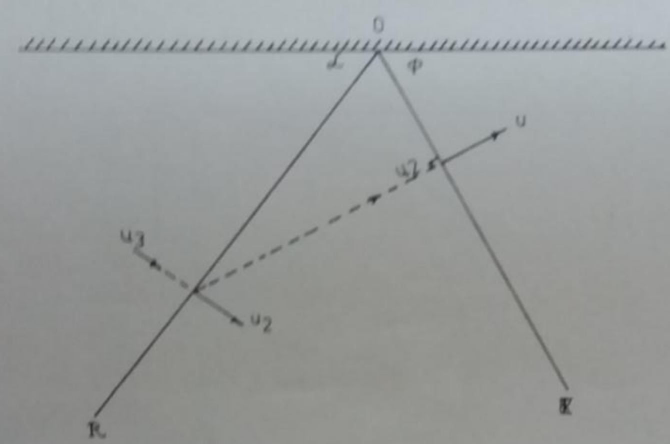
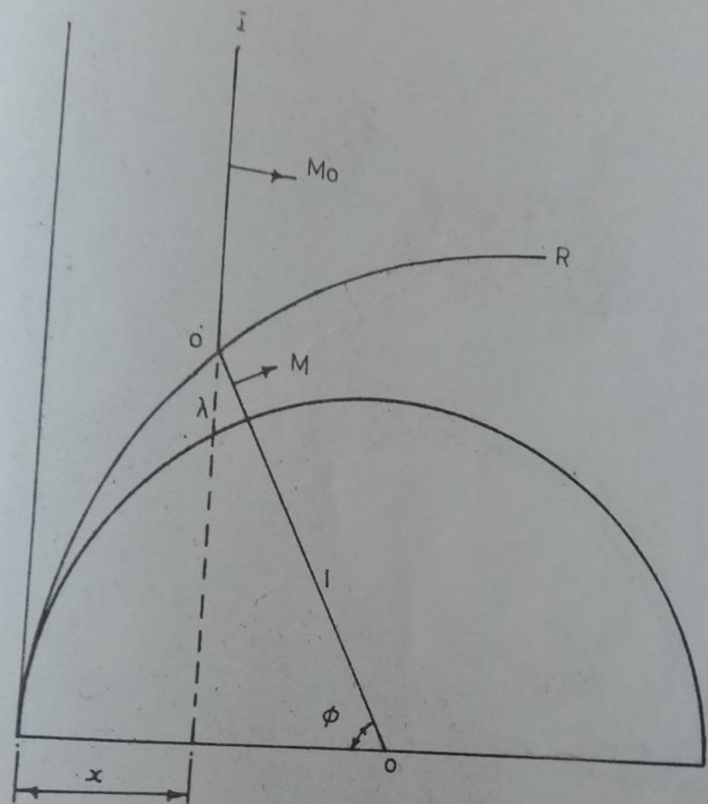


FIG 1-REFLECTION OF PLANE SHOCK FROM A RIGID BOUNDARY



G 2:- INTERACTION OF PLANE SHOCK WITH CIRCULAR CYLINDER

RESTRICTED

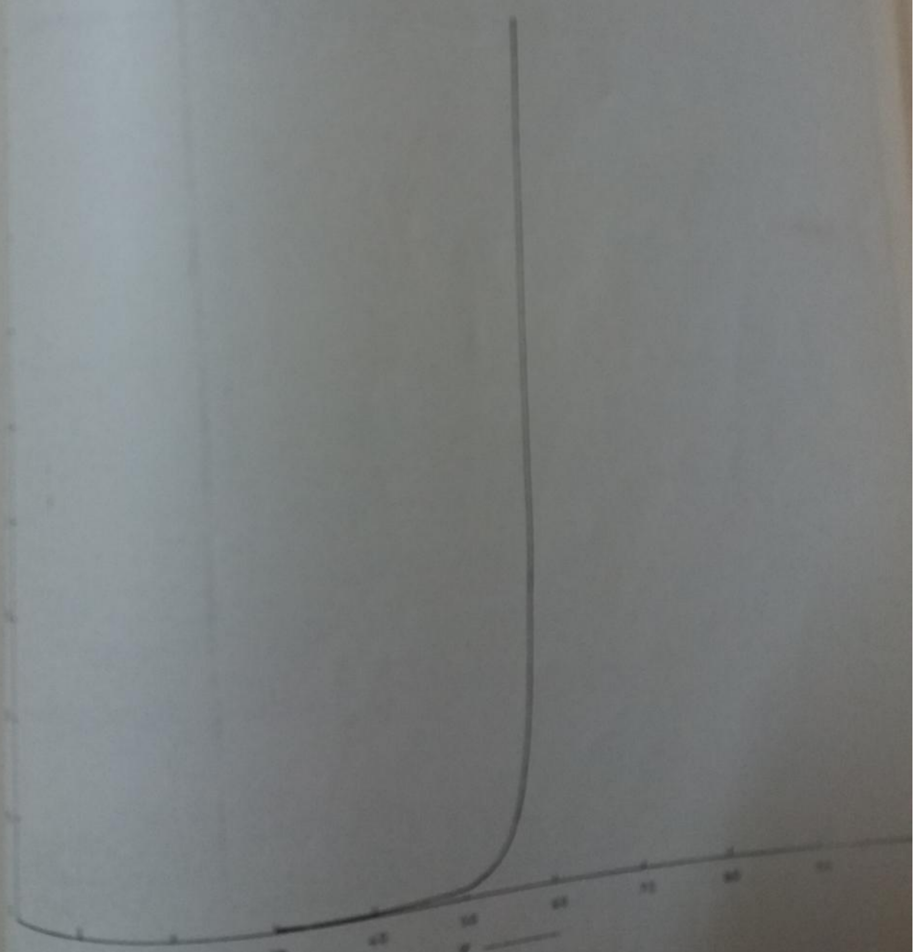
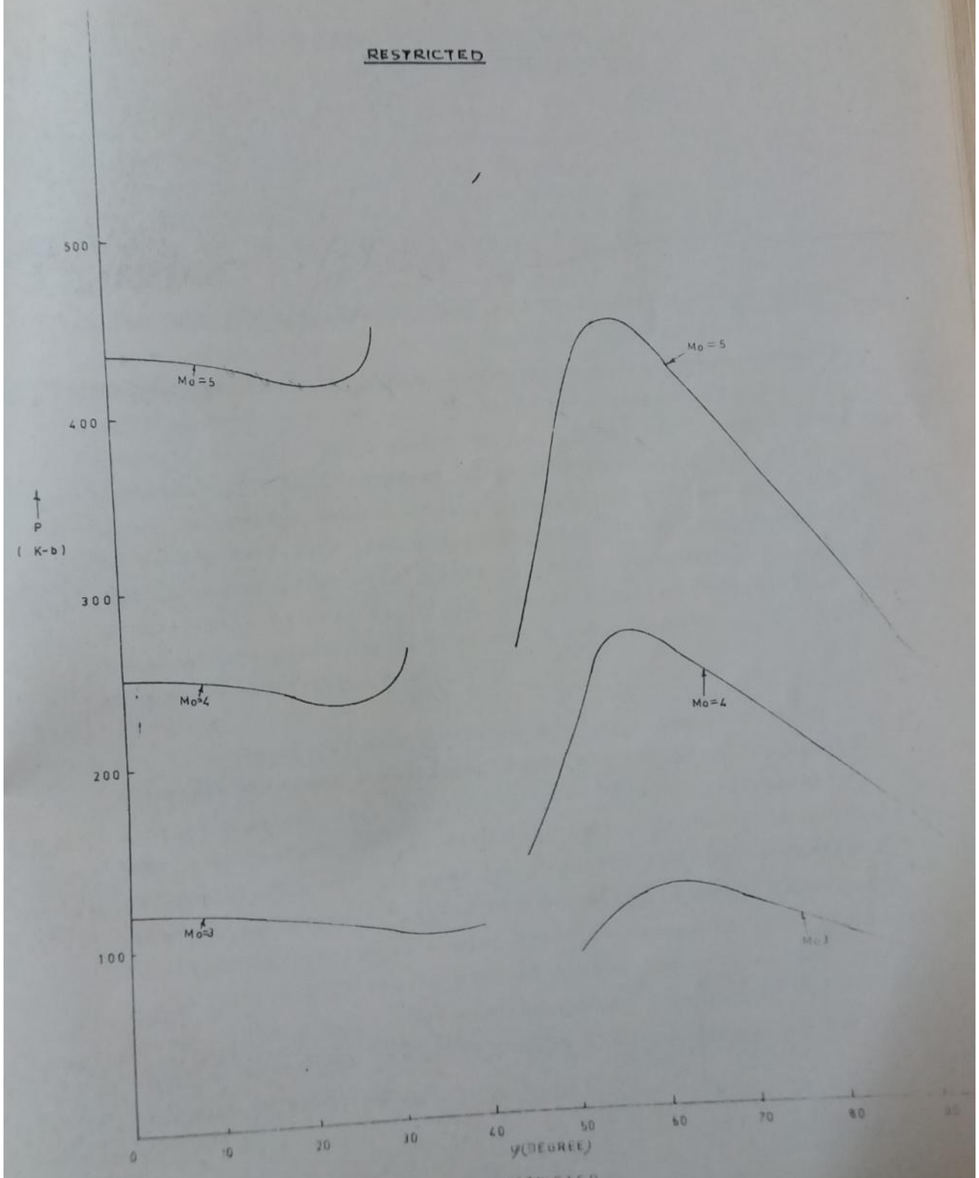


FIG. 3

RESTRICTED

RESTRICTED



RESTRICTED