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Generation of High Pressure and Temperature by Converging Detonation Waves

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ABSTRACT

Generation of high pressure and temperature has various applications in defence. Several techniques, viz flying plate method, collapsing of linear, convergence of detonation waves in solid explosives, have been established in this connection. In the present paper, converging detonation waves in solid explosives, where variable heat of detonation is being added to the front, is studied, by using Whitham's characteristics rule. Results are compared with those reported elsewhere.

1. INTRODUCTION

Study of converging detonation waves in solid explosives is of immense importance due to its applications in the generation of high pressure and temperatures. This problem is studied by various authors, using polytropic equation of state[1-3] (Ref 2 to be referred as paper I). Using solidstate equation of state[4,5] the above problem has been studied[6] and thus variation of pressure and temperature were evaluated during convergence. In these studies[1-6] it was assumed that heat of detonation per unit mass Q remains constant during the process of convergence.

It is argued[7] that imploding detonation wave may be partially or wholly driven by heat released from mechanisms other than chemical reactions. Other mechanisms for the release of heat in a gas such as radiation, conduction and ohmic heating are possible and have the property that the heat released per unit mass is not necessarily constant but is, in general, a function of area of convergence.

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In this paper it is assumed that heat released in chemical reaction depends on the current detonation velocity. Equations of earlier work (Ref 6, here after to be referred as paper II) are modified. It is seen that in the present case increase in radiation pressure and temperature is much higher as compared to that of paper II, where as, there is not much change in total pressure for most of the explosives. No experimental data is available for comparison, only theoretical results are reported.

2. FORMULATION OF PROBLEM

It was assumed in paper II that Q released behind the detonation front remains constant and its value is same as that at C-J plane. But in actual practice it is observed[7] that heat of formation of an explosive depends on various factors such as radiation, conduction and ohmic heating other than chemical reactions. It is also seen that temperature and pressure of explosive increases due to shock compression during convergence and heat released in chemical reaction is a function of temperature and pressure[8] of the products. If the explosive is already at much higher pressure and temperature due to shock compression, the composition of its reactants may change. Also since internal energy of products is higher at higher temperatures, the heat of detonation in this case will be different than that in normal situation. In the present paper we have not gone for all the above factor individually but as a totality of these effects it is assumed that heat of detonation per unit mass Q is variable and is function of current parameter y and U and not of y and D as in paper II. (See Appendix).

Jump conditions in the present case are same as in paper II except in the present case,

$$Q = \frac{[n+(1+\Gamma)y]}{[(n+1)f_1]^2}\left[\frac{1}{n-1}+\frac{y}{\Gamma}-\frac{(1+y)^2}{2[n+(1+\Gamma)y]}\right]U^2 \qquad (1)$$

where all the symbols in the present paper are same as that in II. Solving Eqns. (1) to (3) and (5) to (8) of paper II with the help of Eqn. (1) above, we get two types of solutions given as

Case I

$$U = D\,[z/z_1]^{1/2} \qquad (2)$$

$$u = \frac{(1+\bar{y})\,D}{(n+1)f_1}\,(z\,z_1)^{1/2} \qquad (3)$$

$$\rho/\rho_o = \frac{(n+1)\,f_1}{n\,f_4} \qquad (4)$$

$$p = \frac{\rho_o D^2}{(n+1)}\,z \qquad (5)$$

Case II

$$U = \frac{Dz^{1/2}}{\left[z_1 \left(\frac{2f_1}{1+y} - 1 \right) \right]^{1/2}} \tag{6}$$

$$u = \frac{(1+\bar{y})\, zD}{(n+1)\, \bar{f}_1} \left[z_1 \left(\frac{1+f_1}{1+y} - 1 \right) \right]^{1/2} \tag{7}$$

$$\rho/\rho_o = \frac{(n+1)}{(n-1) f_1 + (1+y)} \tag{8}$$

where

$$z_1 = \frac{(1+y)\ \bar{f}_1}{(1+y) f_1} \tag{9}$$

$$c = \frac{nDf_1}{(n+1) f_1} \left(\frac{z}{z_1} \right)^{1/2} \tag{10}$$

where C is the sound velocity behind detonation front.

$$z = \frac{(n+1)\, P}{\rho_o D^2} = P/P_{cJ} \tag{11}$$

where subscript C-J indicates value of parameters in C-J plane.

It can be shown that jump conditions (6 to 8) give non-overdriven detonations and hence are ignored. Therefore we will take case I only as jump conditions.

In relations (2 to 5) when $y=0$ one gets jump conditions for the case when equation of state is in the polytropic form i.e.

$$U = Dz^{1/2} \tag{12}$$

$$u = \frac{Dz^{1/2}}{(n+1)} \tag{13}$$

$$\rho/\rho_o = \frac{n+1}{n} \tag{14}$$

In the case non-overdriven detonation, we have $z=1$ thus we have

$$U = D/z^{1/2} \tag{15}$$

$$u = \frac{(1+\bar{y})\ D}{(n+1) f_1}^{1/2} \tag{16}$$

$$\rho/\rho_o = \frac{(n+1) f_1}{n f_4} \tag{17}$$

We will deal all these cases in the following section for five different CHON explosives and compare the results with those of paper I and II.

3. CONVERGENCE OF DETONATION WAVES

Following paper II, we use the equation of motion along the positive characteristics axis as an extra relation relating detonation parameters. The characteristic form of the equation of motion is

$$dp + \rho c\, du + \frac{2\rho c^2 u}{u+c}\frac{dR}{R} = 0 \tag{18}$$

where R is the distance of detonation wave from the centre.
Substituting p, ρ, c and u from (3 to 5) and (10) in above relation, one gets after simplifications

$$H_1 \frac{dy}{dR} + H_2 \frac{dz}{dR} + H_3 = 0 \tag{19}$$

where

$$\left.\begin{aligned} H_1 &= \frac{(n-1-\Gamma)}{(1+y)(n+1)f_1} \\ H_2 &= 3/z \\ H_3 &= \frac{4\, n\, f_4}{(n+1)f_1 R} \end{aligned}\right\} \tag{20}$$

From equation of state

$$p = \tilde{c}\,(1+y)\,\rho^n \tag{21}$$

and jump conditions (2 to 5), one gets after differentiation

$$G_1 \frac{dz}{dR} = G_2 \frac{dy}{dR} \tag{22}$$

where

$$G_1 = 1/z \tag{23}$$

$$G_2 = \left[\frac{1}{1+y} + \frac{1}{f_1 f_4}\left\{ \frac{n-1-\Gamma}{n+1} \right\} \right] \tag{24}$$

solving Eqns. (19) and (22) for dy/dR and dz/dr, one gets

$$\frac{dy}{dR} = \frac{-H_3}{(3G_2 + H_1)} \tag{25}$$

$$\frac{dz}{dR} = \frac{-3G_2 H_3}{(3G_2 + H_1)H_2} \tag{26}$$

Equations (25) and (26) are differential equations giving variation of y and z as a function of R. Once y and z are known other functions can be evaluated from jump conditions.

4. DISCUSSION

Equations (25) and (26) are integrated by using Runge-Kutta method of fourth order and results are shown in Figs. 1 to 6. In Fig. 4, it is seen that in the present case y increases continuously while in the case of paper II, y first decreased and then

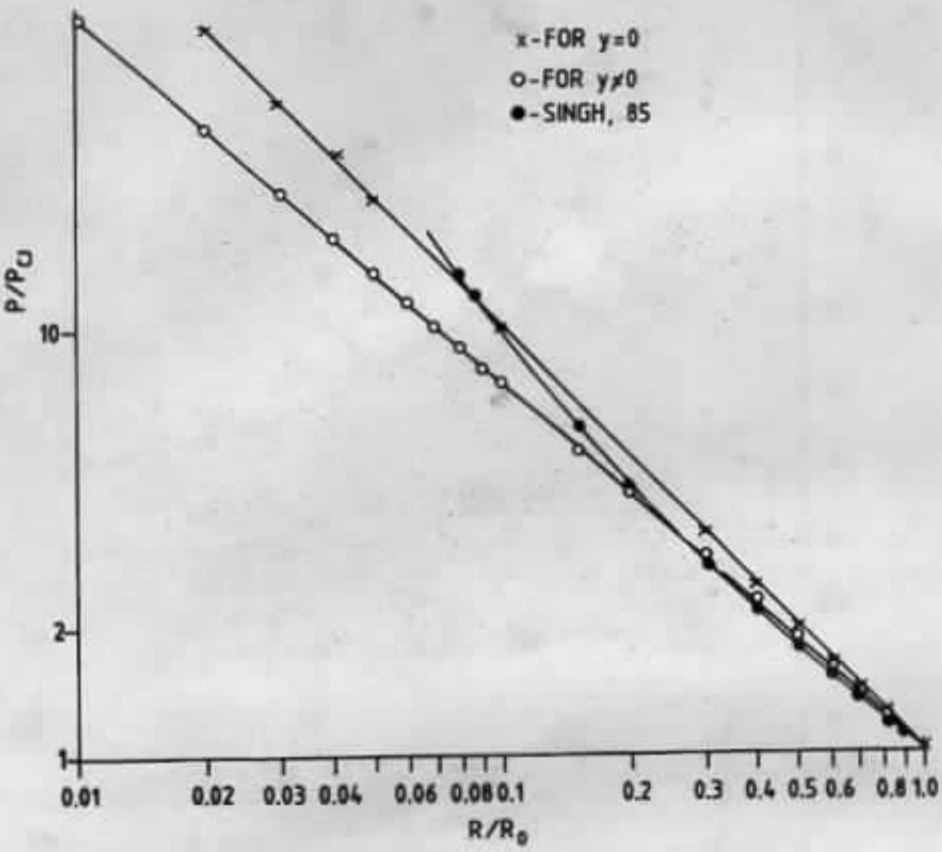

Figure 1. Variation of p/p_{cj} versus R/R_o during convergence in TNT.

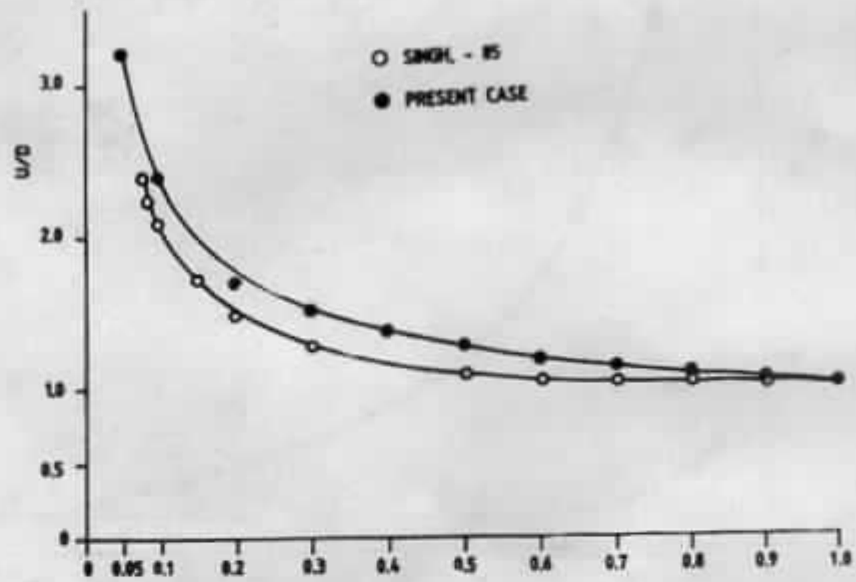

Figure 2. Variation of U/D versus R/R_o during convergence in TNT.

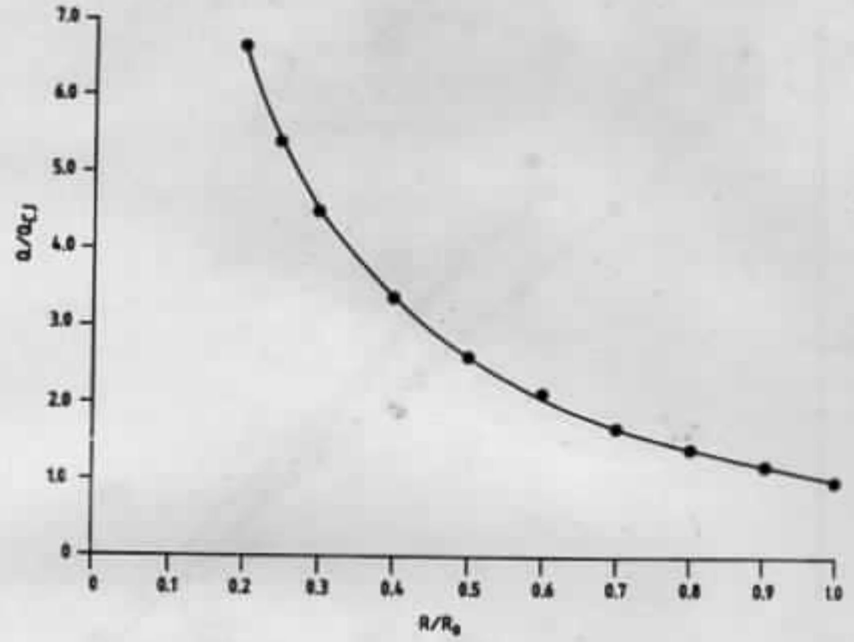

Figure 3. Variation of Q/Q_{cj} versus R/R_o in TNT.

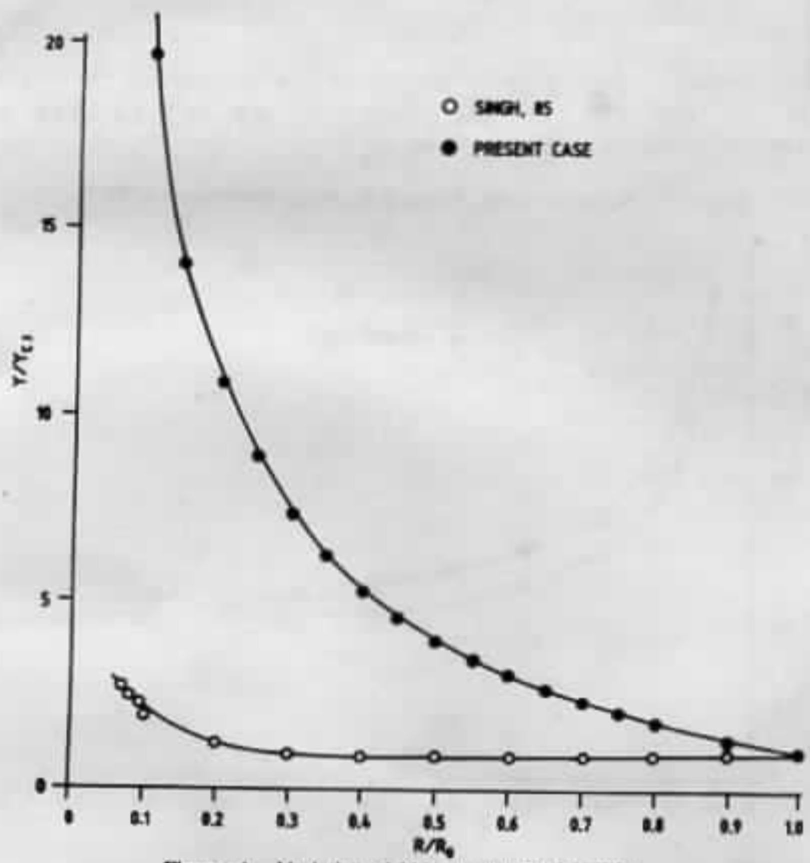

Figure 4. Variation of Y/Y_{cj} versus R/R_o in TNT.

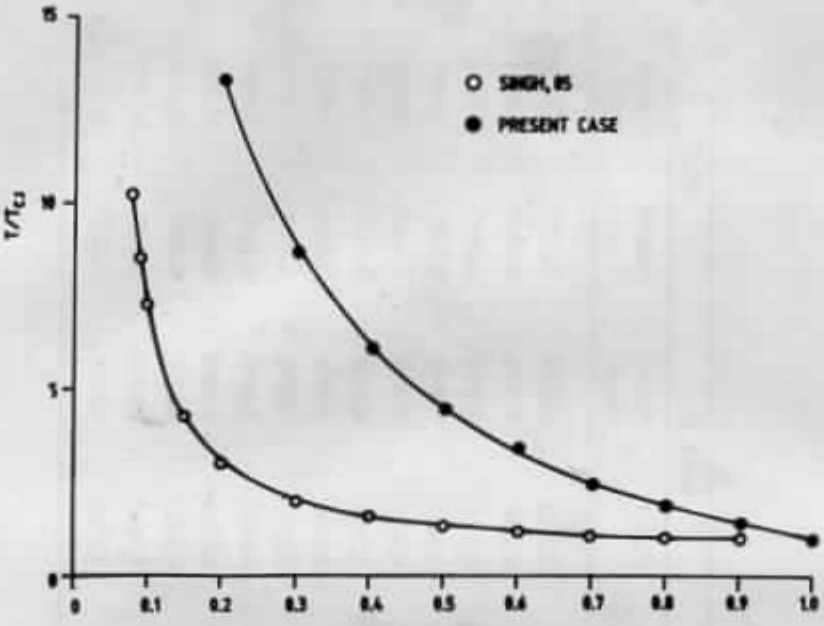

Figure 5. Variation of T/T_{cj} versus R/R_0 in TNT.

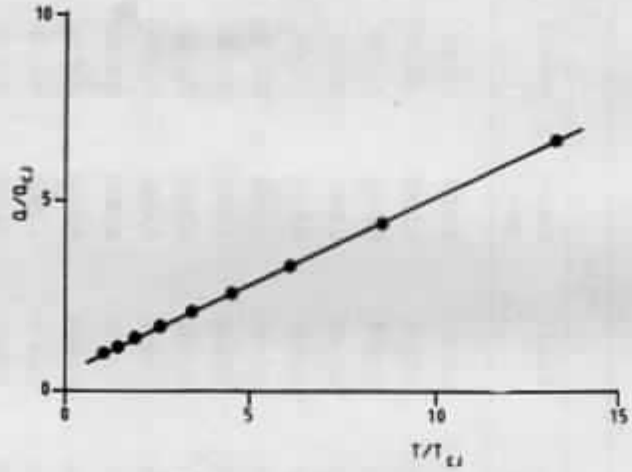

Figure 6. Variation of Q/Q_{cj} versus T/T in TNT.

increased, rate of increase of y in the present case is much higher than that of paper II (Table 1). Pressure in the present case is higher in the beginning but becomes lower after some distance in the case of a few explosives, say, TNT and Tetryl (Table 2). This phenomena is really surprising, as quantity of heat Q continuously increases with convergence (Fig. 3). Effect of Q in this case is mainly on the variation of y i.e. on radiation pressure, which is more in the present case as compared to the earlier case. This radiation pressure is responsible for the increase of temperature in the present case.

Table 1. Variation of Y Vs R/R_o

R/R_o	TNT		PETN		Tetryl		Camp B		RDX	
	Paper II	Present paper	Paper II	Present paper	Paper II	Present paper	Paper II	Present paper	Paper II	Present paper
1.0	0.3757	0.3756	0.2381	0.2381	0.2362	0.2364	0.0784	0.0784	0.2126	0.2126
0.9	0.3653	0.5191	0.2314	0.3662	0.2295	0.3605	0.0759	0.1813	0.2067	0.3381
0.8	0.3558	0.6907	0.2251	0.5189	0.2228	0.5082	0.0735	0.3028	0.2012	2.0974
0.7	0.3470	0.9031	0.2198	0.7051	0.2169	0.6878	0.0713	0.4492	0.1967	2.4142
0.6	0.3413	1.1689	0.2167	0.9389	0.2121	0.9375	0.0695	0.6306	0.1938	2.3145
0.5	0.3402	1.5187	0.2166	1.2446	0.2100	1.2307	0.0686	0.9032	0.1943	3.3327
0.4	0.3478	2.0068	0.2229	1.6680	0.2130	1.6337	0.0693	1.2244	0.2006	4.0657
0.3	0.3740	2.7511	0.2419	2.3076	0.2264	2.2363	0.0737	1.6973	0.2186	5.1813
0.2	0.4487	4.0761	0.2949	3.4320	0.2665	3.2781	0.0870	2.4991	0.2684	7.1584
0.1	0.7419	7.3671	0.4879	6.1978	0.4163	5.7471	0.1322	4.3608	0.4479	12.0593
0.09	0.8181	8.0252	0.5356	6.7413	0.4516	6.2168	0.1422	4.7131	0.4921	12.9335
0.08	0.9223	8.8211	0.5974	7.3963	0.4972	6.7760	0.1547	5.1331	0.5477	14.0881
0.07	1.0377	9.8072	0.6803	8.2043	0.5570	7.4561	0.1700	6.6451	0.6227	15.5133

Table 2 Variation of $Z = (P/P_{cj})$ Vs R/R_o

R/R_o	TNT		PETN		Tetryl		Camp B		RDX	
	Paper II	Present case	Paper II	Present case	Paper II	Present case	Paper II	Present case	Paper II	Present case
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.9	1.0800	1.1003	1.0800	2.6718	1.0800	1.0980	1.0800	4.5258	1.0800	2.6715
0.8	1.1800	1.2237	1.1850	2.9676	1.1800	1.2182	1.1800	5.0073	1.1850	2.9558
0.7	1.3200	1.3796	1.3250	3.3402	1.3200	1.3693	1.3100	6.3881	1.3200	3.3129
0.6	1.5000	1.5832	1.5000	3.8257	1.4900	1.5971	1.4900	7.4324	1.5200	3.7773
0.5	1.7500	1.8616	1.7600	4.4867	1.7400	1.3666	1.7400	8.9310	1.7600	4.3953
0.4	2.1500	2.2673	2.1600	5.4454	2.1400	2.2548	2.1200	11.2825	2.1600	5.3060
0.3	2.8400	2.9187	2.8400	6.9757	2.8000	2.0684	2.7600	15.6043	2.3400	6.7561
0.2	4.3300	4.1566	4.3300	9.8584	4.2100	4.0053	4.1800	26.8634	4.3300	9.4794
0.1	9.9200	7.5714	9.6800	17.6934	9.1600	6.9797	8.6800	29.1469	9.6800	17.4581
0.09	11.4500	8.2904	11.1000	19.3276	10.4000	7.5798	9.7600	31.9189	11.1000	18.2561
0.08	13.600	9.1744	13.000	21.3306	12.0600	8.3063	11.1500	35.3712	12.9500	20.1217
0.07	–	10.2897	15.6500	23.8494	14.3000	9.2057	12.9500	–	15.5500	22.4660

Table 5 Variation of T Vs R/R_0 (T is in Kelvins)

R/R_0	TNT		PETN		Tetryl		Camp B		RDX	
	Paper II	Present case	Paper II	Present case	Paper II	Present case	Paper II	Present case	Paper II	Present case
1.0	2913.76	2913.76	3526.18	3526.18	3925.73	3925.73	3100.32	3100.32	3482.11	3482.21
0.9	3039.16	4111.54	3630.21	5546.23	3546.72	6122.24	3175.32	7342.49	3594.57	5665.09
0.8	3172.13	5589.69	3786.17	8044.77	3680.23	8835.70	3271.82	12578.10	3747.69	41325.97
0.7	3368.06	7475.96	4009.83	11202.94	3866.39	12258.10	3410.33	19173.24	3960.50	48267.84
0.6	3635.01	9906.76	4311.80	15309.74	4143.02	17199.51	3623.75	27719.19	4304.84	57153.11
0.5	4029.74	13196.98	4799.57	20665.85	4558.48	23216.87	3956.81	41206.59	4756.93	68801.31
0.4	4708.18	17913.72	5623.18	28821.61	5284.36	31773.63	4524.85	57890.08	5585.69	85484.12
0.3	5977.13	25294.34	7170.99	41236.31	6603.42	45006.49	5667.82	83622.08	7151.75	111188.47
0.2	8973.73	38758.92	10954.4	63761.93	9735.13	68655.08	8394.82	129428.49	10999.5	157289.29
0.1	21386.6	72973.16	26254.0	–	22113.8	126589.72	18644.9	241564.84	26636.7	272925.64
0.09	24917.8	–	30510.0	–	25368.6	–	21262.3	–	30994.8	–
0.08	29927.5	–	36270.9	–	29751.5	–	24682.7	–	36740.7	–
0.07	–	–	44399.3	–	35800.1	–	29180.6	–	44912.7	–

In the present paper, we have taken Q to be function of y and U, and thus Q is continuously increasing function of U. It is known that chemical reaction depends on temperature and pressure of the reactants and also internal energy of products increases with the temperature. Since the explosive is already at higher temperature and pressure because of shock compression, the total heat liberated by it will obviously be higher.

In the present work, we have not tried to evaluate Q at different temperatures, but as a totality of these factors, it was assumed that Q is a function of current variables y and U instead of C-J parameters y and D.

In Fig. 6, we have plotted Q/Q_{cj} versus T/T_{cj} for the case of TNT. It is seen Q/Q_{cj} increases linearly with T/T_{cj}. In Figs 1, 4 and 5, p/p_{cj}, y/y_{cj} and T/T_{cj} are plotted versus R/R_o for typical explosive TNT, for the present case and results are compared with those reported elsewhere. It is seen (Fig 2) that U/D is higher in the present case as compared to that of paper II where as increase in y/y_{cj} is much higher in the present case (Fig. 4). From Fig. 5, it is seen that increase in the temperature is also much higher than that of earlier case. Variation of y, z and T is shown in Table 1, 2 and 3 respectively for various explosives.

In Fig. 1, we have plotted p/p_{cj} versus R/R_o, for the two cases $y=0$ and $y\neq0$. Results are compared with that of paper II. It is seen that in the present case, p/p_{cj} is higher in the beginning, but becomes little lower near the centre in a few cases, say, TNT and Tetryl.

5. CONCLUSION

It is concluded that the rate of increase of pressure with respect to distance is not as high as that of temperature or in other words the thermal pressure. Thus near the centre, contribution of pressure towards the thermal part is much more than that towards the elastic part.

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REFERENCES

1. Abarbanel, S., *Israel J. Tech.*, **5** (1967), 238-42.
2. Singh, V.P., *J. Ind. Maths. Soci.*, **42** (1978), 387-88.
3. Singh, V.P., *Def. Sci. J.*, **33** (1983), 53-58.
4. Hornberg, H., *Propellants Explosives*, **3** (1978), 97.
5. Zeldovich, J.B. & Kompaneets, A.S., Theory of Detonation (Acad Press, N.Y.), 1960.

6. Singh, V.P., *Pramana*, **24** (1985), 527-35.
7. Strachan, J.D., *Physics Fluids*, **16** (1973), 2020-22.
8. Mayer, R., Explosives (Verlag Chemie, Wheinheen), 1981, pp. 268-76.

APPENDIX

Jump conditions across the detonation front are (from ref 6) (1)

$$\rho(U-u)=\rho_0 U \quad (2)$$

$$\rho(U-u)^2+p=\rho_0 U^2 \quad (3)$$

$$\frac{1}{2}(U-u)^2+E+\frac{p}{\rho}=\frac{1}{2}U^2+Q$$

where E is the internal energy, ρ the density, p the pressure, U the detonation velocity, u, the particle velocity behind the detonation front and the density of unexploded change. Heat of detonation per unit mass, Q is given as

$$Q=[n+(1+\Gamma)y]\left[\frac{1}{(n-1)}+\frac{y}{\Gamma}-\frac{(1+y)^2}{2[n+(1+\Gamma)y]}\right]D^2 \quad (4)$$

where

$$y=\left[\frac{p/\rho}{c_v\,\Gamma T}-1\right] \quad (5)$$

In the relation (5), c_v, Γ, T are respectively the specific heat of constant volume, the Gruineisen constant and the temperature of detonation products. At the surface of charge i.e. of Chapman-Jouguet plane (C-J plane),

$U=D$

$y=\bar{y}$

Here y is defined as the ratio of the thermal pressure to elastic pressure and is given as in ref. 4.

$$y=\frac{c_v\,\Gamma\, v^{n-1}\,T}{\bar{c}} \quad (6)$$

where $\bar{C}$ is defined in ref. 7, v, n respectively are specific volume ($=1/\rho$) and polytropic constant of the explosive.

Total detonation pressure is given as

$$p=\bar{C}\rho^n+\frac{c_v\,\Gamma\, T}{v} \quad (7)$$

and internal energy E is given as

$$E=\frac{p}{(n-1)\rho}\left[1+\frac{(n-1)}{\Gamma}y\right] \quad (8)$$