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Converging shock waves in metals

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Abstract. Study of propagation of a spherically converging shock wave has been carried out by Whitham's method. The variation of shock velocity and pressure along the radius of curvature has been calculated numerically for a number of metals. Attempt has also been made to compare the experimental results of velocity of detonation wave with those reported elsewhere by the application of Whitham's method. A good agreement between experimental and theoretical results has been obtained in this study.

Keywords. Converging shocks; shocks in metals.

1. Introduction

The study of propagation of converging shock waves is of immense importance for the production of very high pressure and temperature. Singh (1978) (hereafter referred to as I) has studied propagation of imploding detonation waves in solid explosives by using Whitham's (1958) as well as Chisnell's (1957) method. Earlier Abarbanel (1967) has also used this method to study propagation of detonation waves in a converging channel, and found detonation velocities numerically.

Following the technique, used in I, an attempt has been made here to study the propagation of spherically converging shocks in various metals. In this analysis, it has been assumed that the shock wave would continue to move with constant velocity if the area of cross-section of the flow does not change. This assumption implies that the parameters of the shock wave are affected only by the convergence in the flow. Further the transmission of shock wave in the spherical metal target has been assumed to take place from spherical explosive pad in which detonation wave has converged to give a pressure higher than that of Chapman Joguet pressure of the explosive. In order to support the contention that Whitham's method is sufficiently accurate for analysing converging shock waves in solids and also to obtain the sufficiently accurate boundary values of shock parameters at metal explosive interface, a comparison of experimental results of velocity of detonation waves with those calculated on the basis of Whitham's method in I has been made.

2. Formulation of the problem

Let us consider a conical section of a solid sphere as the medium for shock wave propagation and assume that a spherical shock wave, having radius of curvature matching the solid sphere, is induced at the base of the cone and it propagates towards its apex without suffering any attenuation due to any rarefaction from the sides or from the base of the cone. If the apex is taken as the origin and R as the radius of curvature, then the equations of conservation of mass, momentum and energy of the medium are given by

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{2\rho u}{r} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0, \tag{2}$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} + c^2\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0, \tag{3}$$

where r indicates the position of shock front at time t and p, ρ, c, and u represent the pressure, density, sound velocity and particle velocity respectively. Further, shock velocity, U and particle velocity, u in the metal have been assumed to be connected by a linear relation

$$U = a + b\,u, \tag{4}$$

where a and b are constants of the metal. If subscript '2' and '1' denote the quantities behind and ahead of the shock front, then mechanical jump conditions across the shock front are given by the expressions

$$p_2 = \frac{\rho_1 a^2 \delta(\delta - 1)}{\{b - \delta(b-1)\}^2}, \tag{5}$$

$$U = a\delta / \{b - \delta(b-1)\}, \tag{6}$$

$$u_2 = a(\delta - 1) / \{b - \delta(b-1)\}, \tag{7}$$

where $\delta = \rho_2/\rho_1$ represents the compression behind the shock front.

A relation between shock radius r and the shock compression δ will be sought, as the solution of the problem, by combining these equations with McGruneisen equation of state of the medium. This relation can be derived by applying Whitham's rule to the characteristics form of equation of motion obtained by linear combination of equations (1) – (3).

3. Solution of the problem

Making a linear combination of equations (1) – (3) one can readily show that the expression for the equation of motion along the positive characteristics is given by

$$\mathrm{d}p + \rho c\,\mathrm{d}u + \frac{2\rho c^2 u}{u + c}\frac{\mathrm{d}r}{r} = 0, \tag{8}$$

where c denotes the adiabatic sound velocity and is related to the slope of the Hugoniot a_H by the expression (Duvall and Fowles 1963)

$$c = a_H (1 - \Delta)^{1/2}, \tag{9}$$

where
$$\Delta = \frac{\Gamma (\delta - 1)}{2} \left[1 - \left(\frac{U - u_2}{a_H} \right)^2 \right], \tag{10}$$

$$a_H = (\partial p_2 / \partial \rho_2)_H.$$

Here Γ is a parameter of McGruneisen equation of state of the medium and is given by the relation (McQueen *et al* 1978)

$$\rho \Gamma = \rho_0 \Gamma_0, \tag{11}$$

where L_0 is the constant of McGruneisen's equation of state at initial density of the metal ρ_0. Differentiating (5), one easily gets the expression for the slope of the Hugoniot as

$$a_H = \frac{a}{[b - \delta (b - 1)]} \left[\frac{\delta + b (\delta - 1)}{\delta - b (\delta - 1)} \right]^{1/2}. \tag{12}$$

Following Singh (1978) as in I, the expressions for dp, du, u and c have been obtained from relations (4)–(6) and (8)–(10) and have been substituted in (7) which, after simplification yields,

$$\frac{2}{r} \frac{\mathrm{d}r}{\mathrm{d}\delta} = k_1 (\delta), \tag{13}$$

where
$$k_1(\delta) = - \left\{ 1 + \frac{1}{\delta - 1} [f/g (1 - \Delta)]^{1/2} \right\} \left\{ 1 + \delta \left[\frac{1 - \Delta}{fg} \right]^{1/2} \right\} \Big/ \delta (1 - \Delta), \tag{14}$$

and
$$f(\delta) = \delta + b (\delta - 1),$$

$$g(\delta) = \delta - b (\delta - 1),$$

$$\Delta = \frac{b \Gamma_0 (\delta - 1)^2}{\delta \{ \delta + b (\delta - 1) \}}. \tag{14a}$$

Defining a dimensionless variable $\xi = r/R$ and knowing initial values of shock parameters at $\xi = 1$, one can readily integrate numerically the differential equation (12) between the limits $\xi = 1$ to $\xi = 0$ to get the shock compression history from base to apex of the conical target.

4. Results and discussion

Initial compression of the metal accomplished by the shock transmitted at $\xi = 1$, has been determined by combining (5) and (6) with the mismatch equation

$$\frac{P_2}{P_D} = \frac{2 \rho_1 U}{\rho_D U_D + \rho_1 U}, \tag{15}$$

where U_D and P_D denote the velocity and pressure of the detonation wave and ρ_D represents the initial density of the explosive.

In this analysis velocity and pressure of the detonation wave, used in (12), have been obtained from I for a detonation wave which has converged over a thickness of 5 cm of the explosive. The explosive pad was concentric with the spherical boundary of the metallic cone. The use of mismatch equation for computing transmitted initial pressure in this case, therefore, implies an assumption that this equation holds good for overdriven detonation waves with good approximation. The initial values of shock pressure and velocity transmitted by 5 cm thick RDX/TNT (60:40) explosive pad in different metals have been presented in table 1.

To ascertain the correctness of these initial values the velocity of a converging detonation wave has also been determined experimentally at different points along the thickness of the pad by a method similar to that of Cheret and Verdes (1970). These experimentally determined values of detonation velocity in RDX/TNT explosive of density 1·68 gm/cc have been fitted by least square method in an equation.

$$U_D = 7{\cdot}8 + 1{\cdot}4164\, x + {\cdot}01828\, x^2 \ldots, \tag{16}$$

where x is the distance in cm and U_D is the detonation velocity in km/sec. The distance x has been measured along the radius from the outer surface of the spherical pad of the explosive.

The experimental velocities obtained from (16) have been compared with those of I and were found in good agreement, along the entire thickness of the explosive, as shown in table 1.

Table 1. Transmitted shock parameters obtained from theoretical and experimental detonation velocity. The detonation wave was allowed to converge spherically over a thickness of 5 cm in an explosive pad of RDX/TNT (60 : 40) before transmitting a shock wave in the metal. The constants a and b appear in the linear relation $U = a + b\, U_p$.

Name of the metal	a	b	Shock parameters from theoretical detonation velocity		Shock parameters from experimental detonation velocity	
			δ	U	δ	U
Aluminium	5·328	1·338	1·383015	8·464511	1·367748	8·32174
Copper	3·940	1·489	1·310918	6·091105	1·297710	5·984157
Beryllium	7·998	1·124	1·303647	10·834512	1·290010	10·702372

Computer has been used to integrate (12) numerically by Runge Kutta method. The values of constants of this equation have been taken from Kinslow (1970). The increment of compression δ used in the computation was 0·005. The convergence of shock wave in aluminium, copper and beryllium has been studied, between $\xi = 1$ to $\xi = 0$.

The variations of shock pressure with dimensionless radius of convergence ξ, thus obtained, have been shown in figure 1. The effect of variation in Γ on pressure distance curve is shown in figure 2, for two cases *i.e.* $\Gamma = \Gamma_0$ and $\Gamma = \Gamma_0/8$. Further the effect of initial compression on convergence has also been studied and shown in figure 3. It depicts the variation of shock pressure of a converging wave along dimensionless radius ξ for the initial compression varying from 1·1 to 1·6.

5. Conclusion

Pressure distance curves in figure 1 show that the pressure of spherically converging shock wave increases according to a power law

$$Pr^k = \text{constant}, \tag{17}$$

where K is the convergence factor of the metal. Its average value for different metals is given in table 2. It is interesting to note that in the case of converging shocks in

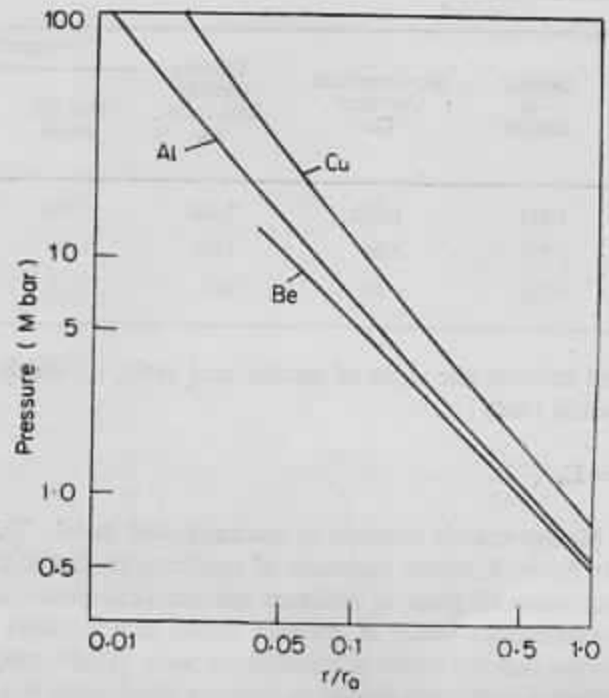

Figure 1. Variation of shock pressure p with dimensionless radius of shock wave $\xi = r/R$ in aluminium, beryllium and copper

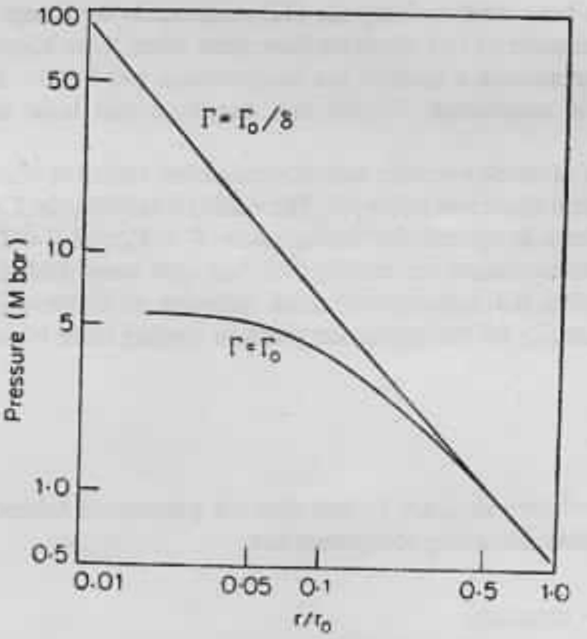

Figure 2. Comparison of shock pressure attenuation for two cases $\Gamma = \Gamma_0$ and $\Gamma = \Gamma_0/\delta$.

Table 2. Comparison of convergence factors for spherically converging shock wave in metals and gases.

Metal	Density ρ_0 gm/cm^3	Mc Gruneisen Constant Γ_0	Effecive specific heat ratio γ_m	Convergence factor, K	
				For the metal	For a gas of specific heat ratio $\gamma = \gamma_m$
Be	1·851	1·160	2·160	0·991	0·991
Al	2·785	2·00	3·00	1·10	1·10
Cu	8·950	1·99	2.99	1·13	1·10

metals, the metal behaves like a gas of specific heat ratio, γ_m which is given by the relation (Zeldowitch 1966)

$$\gamma_m = \Gamma_0 + 1 \tag{18}$$

where Γ_0 is the McGruneisen's constant of uncompressed metal. This fact has been clearly shown in figure 4, where variation of convergence factor for a spherically converging shock wave in gases of different specific heat ratios has been plotted along with the convergence factor of different metals as a function of specific heat ratios, γ_m. It shows that the convergence of shock wave in lighter metals like beriyllium and aluminium is very much similar to that of a shock wave in a gas throughout the range of convergence considered here but heavier metals like copper show a little

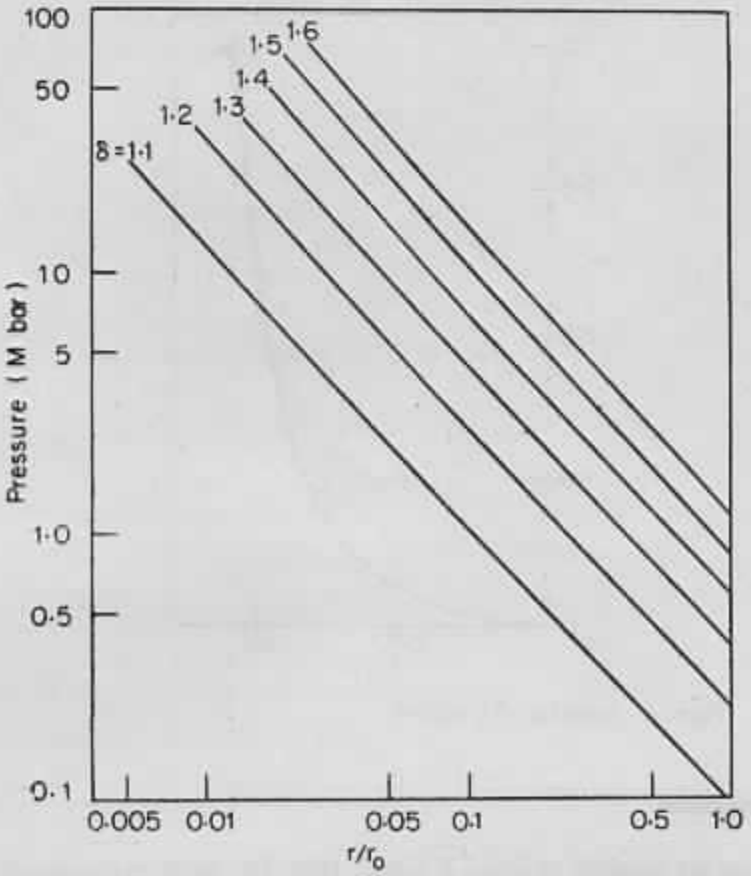

Figure 3. Variation of shock pressure in aluminium for different initial transmitted pressures.

variation from gas-like behaviour particularly in the higher range of convergence. Variation of pressure with the variable Γ is compared with that of constant Γ, in figure 2.

The plot of shock pressure and distance in figure 3 reflects that the shock pressure increases monotonically as it moves towards the centre of convergence. In a particular metal, the rate of increase in the shock pressure with the distance travelled remains unaffected with the initial value of the shock pressure transmitted at the base of the metallic cone. These curves indicate an infinite pressure of the shock wave near the centre as it presents a point of singularity in this analysis.

The fact that the experimental values of detonation velocity and compression are in good agreement with the theoretical values obtained earlier by the present method of analysis, suggest that the Whitham rule of characteristics and mismatch equation holds good for analysing spherically converging shock waves with adequate accuracy upto sufficiently near to the centre of convergence.

It is interesting to note from (13) that the variation of shock compression along the radius of curvature does not explicitly depend on any material property of the metal except the McGruneisen constant Γ, and the constant b of (4).

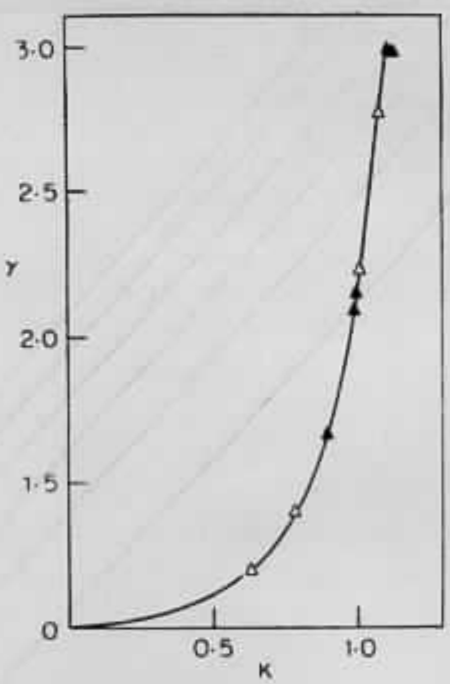

Figure 4. Variation of K versus γ.

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