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STRUCTURE OF SPHERICAL BLAST WAVES IN AIR

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Abstract

Analytical relations for flow behind blast waves are derived using perturbation technique. The equations developed are of general nature and can be applied to different types of equations of state. The solutions obtained in the case of diatomic air are compared with those obtained by Random-Choice Mehtod reported elsewhere.

1. Introduction

The study of the structure of blast waves is quite a complex phenomena involv- ing, hot ionized gasses, rarefaction waves, contact discontinuities headed by a shock front Theoretical and experimental studies of blast waves were conducted by various authors, vi McFadden[1], Brode[2], Chisnell[3] Whitham[4] and Friedman[5]. These studies in genera have shown that blast wave consists of main shock followed by a contact discontinuity an rarefaction wave, entering into high pressure gas at the centre. Blast wave in laborator can be produced by the release of very high pressure gas in the air, but in actual practic detonation of high explosive in air will produce a blast wave.

MacFadden used perturbation method to study the structure of blast wave. Fir order results obtained by him were showing few discontinuities at the boundaries. Modif ing the boundary conditions, these discontinuities have been smoothened out. The partic velocity, sound speed and entropy are developed in series of y which is proportional to tim The zero-order solutions are plane shock-tube solutions. First, second and third order cc rections are derived for various regions. Solutions are matched with the boundary conditio at the boundaries of each region and a process for evaluating the integrating constants outlined.

2. Mathematical model

A perfect gas at high pressure is held stationary within a sphere of unit radi surrounded by air. At time t=0, there is an explosion and the gases rush outward compress the air around it. The subsequent behavior may be described by a space-time diagram a

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Fig 1. Region A contains undisturbed gas. Region B (bounded by a "head" and a "tail") is a rarefaction wave, in which gas is rapidly expanding irrespective of the behaviour outside the tail. Region C consists of rarefied gas which is moving outward. A contact surface or an interface separates the gas from region D, which contains compressed air also moving outward. The transition from the undisturbed air in region E into region D takes the form of a shock[1,4]. Following MacFadden, perturbation method is used to find the flow behind the shock. The basic equations of motion for the spherical flow of an inviscid fluid in the absence of heat conduction are

$$u_t + uu_x + \mu cc_x - c^2 s_x = 0, \tag{1}$$

$$\mu(c_t + uc_x) + c(u_x + \frac{2u}{x}) = 0, \tag{2}$$

$$s_t + us_x = 0, \tag{3}$$

where u, c and s are non-dimensional particle velocity, sound speed and entropy. Subscripts t and x denote the partial differentiation, with respect to time and space. The pressure and density in terms of c and s are given by

$$p = c^{\mu+2} \ . \ e^{-\gamma s}, \qquad \rho = \gamma c^{\mu} \ . \ e^{-\gamma s}, \tag{4}$$

where $s = \frac{S}{C_v\gamma(\gamma-1)}$ = non-dimensional entropy function and $\mu = (2N-1)$, $N = \frac{\gamma+1}{2(\gamma-1)}$, not necessarily an integer, $\gamma = \frac{C_p}{C_v}$, C_p = specific heat capacity per mole at constant pressure, C_v = specific heat capacity per mole at constant volume. Equations (1) to (3) must be satisfied in each of the regions A, B, C, D and E in Fig 1, but the solution in regions A and E are trivial. The entropy scale has been choosen so that S=0, when p=1 and $\rho = \gamma$.

In region A, if we take the initial conditions as u = 0, $c = c_A$, $s = s_A$, (c_A and s_A being constants) in the gas where $0 < x < 1$, then the solutions of (1) to (3) in region A are given by u = 0, c = 1, s = 0. These solutions hold good in region E also, provided that the boundary conditions between A and E are satisfied.

2.1 Approximation for small time

To study the initial behaviour of a spherical blast, we define parameters q and y as

$$q = \frac{1}{\mu+1}[\mu + \frac{1-x}{y}], \quad y = c_A t. \tag{5}$$

Here q is the linear function of the slope $\frac{dt}{dx}$ of a straight line given by $x = 1 - c_A t$ which passes through x=1 and t=0 (see Fig 1). The transformation (5) has two advantages: the singularity at x=1, t=0 being removed, and the boundary conditions are approximately independent of y. We assume u(q,y), c(q,y) and s(q,y) as regular functions of q and y in each of the regions B, C, D. We attempt a power series expansion in powers of y, assuming that the head of the rarefaction wave has moved a small portion of the initial gas radius and $c_A t << 1$, as

$$u(q,y) = \sum_{n=0}^{\infty} u_n(q) y^n, \quad c(q,y) = \sum_{n=0}^{\infty} c_n(q) y^n, \quad s(q,y) = \sum_{n=0}^{\infty} s_n(q) y^n. \qquad (6)$$

The transformation (5) and power series (6) are substituted in (1) to (3) and coefficients of $y^0, y, y^2 ... y^n$ are compared to get zeroth, first, ... nth order approximation equations as follows.

Zeroth order approximation

$$[c_A\lambda - u_o]u_o' - \mu c_o c_o' + c_o^2 s_o' = 0, \qquad (7)$$

$$-c_o u_o' + \mu[c_A\lambda - u_o]c_o' = 0, \qquad (8)$$

$$[c_A\lambda - u_o]s_o' = 0, \qquad (9)$$

where ($'$) denotes the differentiation with respect to q.

First order approximation

$$[c_A\lambda - u_o]u_1' + (2c_A N - u_o')u_1 - \mu c_o c_1' + [2c_o s_o' - \mu c_o']c_1 + c_o^2 s_1' = 0, \qquad (10)$$

$$-c_o u_1' - \mu c_o' u_1 + \mu[c_A\lambda - u_o]c_1' + [2c_A N\mu - u_o']c_1 = -4N c_o u_o, \qquad (11$$

$$-s_o' u_1 + [c_A\lambda - u_o]s_1' + 2c_A N s_1 = 0. \qquad (1$$

The n^{th} order approximation ($n \geq 2$).

$$[c_A\lambda - u_o]u_n' + (2nNc_A - u_o')u_n - \mu c_o c_n' + [2c_o s_o' - (2N-1)c_o']c_n$$
$$+c_o^2 s_n' - \sum_{i=1}^{n-1} u_i' u_{n-i} + \sum_{i=1}^{n-1}[2c_o s_{n-i}' + 2c_1 s_{n-i-1}' - \mu c_{n-i}']c_i - c_1^2 s_{n-2}' = 0, \qquad ($$

$$-c_o u_n' - \mu c_o' u_n - \mu \sum_{i=1}^{n-1} c_i' u_{n-i} + \mu[c_A\lambda - u_o]c_n' + [2nNc_A\mu - u_o']c_n$$
$$- \sum_{i=1}^{n-1} u_i' c_{n-i} + 4N \sum_{j=0}^{n-1}(-1)^j \lambda^j (\sum_{i=j}^{n-1} c_{i-j} u_{n-i-1}) = 0,$$

$$[c_A\lambda - u_o]s_n' + 2nNc_A s_n - \sum_{i=1}^{n} u_i s_{n-i}' = 0,$$

where $\lambda = (2N - 1 - 2Nq)$.

3.0 Boundary conditions

The initial conditions in region A which is an undisturbed gas are given

$$u = 0, \qquad c = c_A, \qquad s = s_A.$$

Here and in the following text subscripts A, B, C and D refer to the variou

3.1 Conditions at the head

The head of the rarefaction wave travel inward at the constant speed of sound according to the equation $x_H = 1 - c_A t$. This is the path of characteristic of differential equations (1) to (3), discontinuties in the derivatives of u and c are permissible at the head but u, c and s must be continuous at $x = x_H$ or $q = 1$. Therefore the zeroth and higher order boundary conditions at the head are

$$u_o(1) = 0, \quad c_o(1) = c_A, \quad s_o(1) = s_A \tag{16}$$

$$u_n(1) = 0, \quad c_n(1) = 0, \quad s_n(1) = 0, \quad for \;\; n = 1, 2, 3, \cdots. \tag{17}$$

3.2 Conditions at the tail

The tail of the rarefaction wave moves at variable speed always inward relative to the particles. For a plane shock, it is the characteristic path. We tried to extend this condition in this case for every order of approximation. If the tail is a characteristic, its slope is

$$\frac{dx}{dt} = u - c.$$

At $x = x_T$ and q at the tail varies with y given by

$$\frac{dx_T}{dt} = u(q_T) - c(q_T) \tag{18}$$

$$q = q_T = q_{T_o} + q_{T_1} y + q_{T_2} y^2 + q_{T_3} y^3 + \cdots.$$

Expanding (18) in terms of q and y and comparing the various powers of y we get,

$$q_{T_o} = \frac{[-\{u_o(q_{T_o}) - c_o(q_{T_o})\} + c_A \mu]}{2N c_A}, \qquad q_{T_n} = -\frac{[u_n(q_{T_o}) - c_n(q_{T_o})]}{2 c_A N} \quad n = 1, 2, \cdots. \tag{19}$$

Since the disturbance at $q = q_T$ is moving inward at sonic speed relative to the particles, the zero order condition is given by (5) as

$$u_o(q_T) - c_o(q_T) = c_A(\mu - 2N q_T) \quad -?$$

$u_0(q_{T_0}), c_0(q_{T_0}$

and the higher order conditions are

i.e. We consider expansion

$$[u_i(q_T) - \mu c_i(q_T)]_B = [u_i(q_T) - \mu c_i(q_T)]_C \quad i = 1, 2, 3 \cdots. \quad (?) \tag{20}$$

$$s_o(q_T) = s_A, \quad s_i(q_T) = 0; \quad i = 1, 2, 3 \cdots. \tag{21}$$

3.3 Conditions at the interface

The interface between the gas and the air moves outward at variable speed. This is a particle path, the velocity must be continuous across this path. Since the acceleration of the interface is finite, the pressure must also be same on both the sides except when t=0. Therefore the boundary conditions at the interrface $x = x_I$ are given by

$$u_C = u_D, \quad t \geq 0; \quad p_C = p_D, \quad t > 0.$$

At $x = x_I$, q and u are given by

$$q = q_I = q_{I_0} + q_{I_1}y + q_{I_2}y^2 + q_{I_3}y^3 + \cdots,$$
$$\frac{dx_I}{dt} = u(q_I). \tag{22}$$

Expanding (22) in terms of powers of y and comparing various powers, we get

$$q_{I_0} = \frac{[-u(q_{I_0}) + c_A\mu]}{2Nc_A}, \quad q_{I_n} = -\frac{u_n(q_{I_0})}{(2n+2)Nc_A} \;; \quad n = 1, 2, 3 \cdots, \tag{23}$$

The boundary conditions for various orders of approximation are given by

$$[u_n]_C = [u_n]_D; \quad [p_n]_C = [p_n]_D; \quad n = 0, 1, 2, 3 \cdots, \quad (2$$

3.4 Conditions at the shock front

The shock front moves outward into the air at a variable velocity U. The sh conditions in terms of non-dimensional parameter U are given by Rankine-Hugoniot e tions,

$$u_D = \frac{\mu(U^2 - 1)}{2NU},$$
$$c_D^2 = \frac{[(\mu + 2)U^2 - 1][U^2 + \mu]}{4N^2U^2},$$
$$p_D = \frac{(\mu + 2)U^2 - 1}{2N}.$$

At $x = x_s$, q and U can be written as

$$q = q_S = q_{s_0} + q_{s_1}y + q_{s_2}y^2 + q_{s_3}y^3 + \cdots,$$
$$U = U_0 + U_1y + U_2y^2 + U_3y^3 + \cdots,$$
$$U = \frac{dx_s}{dt}.$$

Evaluation of $q_{s_0}, q_{s_1} q_{s_2}, q_{s_3}$ ... at the shock front, are similar to that of q_{I_0}, q_I at the interface. They are given by

$$q_{S_0} = \frac{[-U_0 + c_A\mu]}{2Nc_A}, \quad q_{S_n} = -\frac{U_n}{(2n+2)Nc_A} \;; \quad n = 1, 2, 3 \cdots$$

4.0 Solutions in region B ($1 \geq q > q_T$)

4.1 Zero-order solutions

From equation (9) and the boundary conditions (16) we get $s_o = s_A$. Solving (7) and (8) we get for the rarefaction wave following solutions which are compatible with boundary conditions

$$u_o = A_{oo} + A_{01} q, \qquad c_o = B_{oo} + B_{01} q, \qquad s_o = C_{oo} + C_{01}\, q, \tag{29}$$

where

$$\begin{aligned} A_{oo} &= c_A \mu, & A_{o1} &= -c_A \mu \\ B_{oo} &= 0, & B_{01} &= c_A, \\ C_{oo} &= s_A, & C_{01} &= 0. \end{aligned}$$

4.2 First order solutions

Solving (10) to (12) together with boundary conditions (17) at the head, we get first order solutions for N$\neq$ 1,2 as:

$$\begin{aligned} u_1 &= A_{11}\, q + A_{12}\, q^2 + A_{13}\, q^N, \\ c_1 &= B_{11}\, q + B_{12}\, q^2 + B_{13}\, q^N, \\ s_1 &= 0, \end{aligned} \tag{30}$$

where

$$\begin{aligned} A_{11} &= -c_A \mu/(N-1), \\ A_{12} &= 1.5 c_A \mu/(N-2), \\ A_{13} &= -c_A \mu(N+1)/[2(N-1)(N-2)] \\ B_{11} &= -c_A \mu/(N-1), \\ B_{12} &= c_A(4N-3)/[2(N-2)], \\ B_{13} &= -c_A(3N-1)/[2(N-1)(N-2)]. \end{aligned}$$

4.3 Second order solutions

Solving equations (13) to (15), for n = 2 together with the boundary conditions (17) at the head, we find the following solutions for the second order coefficients in region B, for $N \neq 0, \frac{1}{2}, 1, \frac{3}{2}, 2$;

$$u_2 = A_{21}q + A_{22}q^2 + A_{23}q^3 + A_{24}q^N + A_{25}q^{N+1} + A_{26}q^{2N-1} + A_{27}q^{2N},$$
$$c_2 = B_{21}q + B_{22}q^2 + B_{23}q^3 + B_{24}q^N + B_{25}q^{N+1} + B_{26}q^{2N-1} + B_{27}q^{2N}, \qquad (31)$$
$$s_2 = 0,$$

where

$$
\begin{aligned}
A_{21} &= [-4Na_{11} + 2a_{21}]/[12c_A N\mu],\\
A_{22} &= [-(4N-1)a_{12} + 3a_{22}]/[12c_A N(2N-2)],\\
A_{23} &= [-(4N-2)a_{13} + 4a_{23}]/[12c_A N(2N-3)],\\
A_{24} &= [-(3N+1)a_{14} + (N+1)a_{24}]/[12c_A N^2],\\
A_{25} &= [-3Na_{15} + (N+2)a_{25}]/[12c_A N(N-1)],\\
A_{26} &= [-(2N+2)a_{16} + 2Na_{26}]/[12c_A N],\\
A_{27} &= -[A_{21} + A_{22} + A_{23} + A_{24} + A_{25} + A_{26}],\\
B_{21} &= t_1A_{21} + t_2(a_{11} + a_{21}),\\
B_{22} &= t_1A_{22} + t_2(a_{12} + a_{22}),\\
B_{23} &= t_1A_{23} + t_2(a_{13} + a_{23}),\\
B_{24} &= t_1A_{24} + t_2(a_{14} + a_{24}),\\
B_{25} &= t_1A_{25} + t_2(a_{15} + a_{25}),\\
B_{26} &= t_1A_{26} + t_2(a_{16} + a_{26}),\\
B_{27} &= t_1A_{27},\\
a_{11} &= -\{A_{11}^2 + \mu B_{11}^2\},\\
a_{12} &= -3\{A_{11}A_{12} + \mu B_{11}B_{12}\},\\
a_{13} &= -2\{A_{12}^2 + \mu B_{12}^2\},\\
a_{14} &= -(N+1)\{A_{11}A_{13} + \mu B_{11}B_{13}\},\\
a_{15} &= -(N+2)\{A_{12}A_{13} + \mu B_{12}B_{13}\},\\
a_{16} &= -N\{A_{13}^2 + \mu B_{13}^2\},\\
a_{21} &= 4Nc_A^2\mu^2 + 2NA_{11}B_{11} - 4Nc_A\mu B_{11},\\
a_{22} &= -4Nc_A[A_{11} + \mu\{c_A(4N-1) + B_{12} - B_{11}\}]\\
&\quad + A_{11}B_{12} + 2A_{12}B_{11} + \mu(A_{12}B_{11} + 2A_{11}B_{12}),\\
a_{23} &= 8N^2c_A^2\mu + 4NA_{12}B_{12} - 4Nc_A\{A_{12} - \mu B_{12}\},\\
a_{24} &= -4Nc_A\mu B_{13} + (A_{11}B_{13} + NA_{13}B_{11}) + \mu(A_{13}B_{11} + NA_{11}B_{13})
\end{aligned}
$$

$$a_{25} = 4Nc_A\{-A_{13} + \mu B_{13}\} + (2A_{12}B_{13} + NA_{13}B_{12}) + \mu(2A_{13}B_{12} + NA_{12}B_{13}),$$
$$a_{26} = 2N^2 A_{13}B_{13},$$
$$t_1 = (4N-1)/[\mu(\mu+2)],$$
$$t_2 = 1/[2c_A\mu(\mu+2)],$$

where A_{27} is an integrating constant.

4.4 Third order solutions

Solving (13) to (15), for n =3 together with the third order boundary conditions (17), we find the following solutions for the third order coefficients, in this region for $N \neq -1, 0, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 1, \frac{4}{3}, \frac{3}{2}, 2$:

$$u_3 = A_{31}q + A_{32}q^2 + A_{33}q^3 + A_{34}q^4 + A_{35}q^N + A_{36}q^{N+1} + A_{37}q^{N+2}$$
$$+A_{38}q^{2N-1} + A_{39}q^{2N} + A_{310}q^{2N+1} + A_{311}q^{3N-2} + A_{312}q^{3N-1} + A_{313}q^{3N},$$
$$c_3 = B_{31}q + B_{32}q^2 + B_{33}q^3 + B_{34}q^4 + B_{35}q^N + B_{36}q^{N+1} + B_{37}q^{N+2} \qquad (32)$$
$$+B_{38}q^{2N-1} + B_{39}q^{2N} + B_{310}q^{2N+1} + B_{311}q^{3N-2} + B_{312}q^{3N-1} + B_{313}q^{3N},$$
$$s_3 = 0,$$

where

$$A_{31} = [6Na_{31} + a_{32}]/8Nc_A(3N-1),$$
$$A_{32} = 3[(6N-1)a_{33} + a_{34}]/16Nc_A(3N-2),$$
$$A_{33} = 4[(6N-2)a_{35} + a_{36}]/16Nc_A(3N-3),$$
$$A_{34} = 5[(6N-3)a_{37} + a_{38}]/16Nc_A(3N-4),$$
$$A_{35} = (N+1)[(5N+1)a_{39} + a_{310}]/32N^2c_A,$$
$$A_{36} = (N+2)[5Na_{311} + a_{312}]/16Nc_A\mu,$$
$$A_{37} = (N+3)[(5N-1)a_{313} + a_{314}]/16Nc_A(2N-2),$$
$$A_{38} = 2N[(4N+2)a_{315} + a_{316}]/16Nc_A(N+1),$$
$$A_{39} = (\mu+2)[(4N+1)a_{317} + a_{318}],$$
$$A_{310} = (2N+2)[4Na_{319} + a_{320}]/16Nc_A(N-1),$$
$$A_{311} = (3N-1)[(3N+3)a_{321} + a_{322}]/32Nc_A,$$
$$A_{312} = 3N[(3N+2)a_{323} + a_{324}]/16Nc_A,$$
$$A_{313} = -(A_{31}+A_{32}+A_{33}+A_{34}+A_{35}+A_{36}+A_{37}+A_{38}+A_{39}+A_{310}+A_{311}+A_{312}),$$
$$B_{31} = t_3A_{31} - t_4[2a_{31} - a_{32}],$$
$$B_{32} = t_3A_{32} - t_4[3a_{33} - a_{34}],$$
$$B_{33} = t_3A_{33} - t_4[4a_{35} - a_{36}],$$
$$B_{34} = t_3A_{34} - t_4[5a_{37} - a_{38}],$$

$$B_{35} = t_3 A_{35} - t_4[(N+1)a_{39} - a_{310}],$$
$$B_{36} = t_3 A_{36} - t_4[(N+2)a_{311} - a_{312}],$$
$$B_{37} = t_3 A_{37} - t_4[(N+3)a_{313} - a_{314}],$$
$$B_{38} = t_3 A_{38} - t_4[2N a_{315} - a_{316}],$$
$$B_{39} = t_3 A_{39} - t_4[(\mu+2)a_{317} - a_{318}],$$
$$B_{310} = t_3 A_{310} - t_4[(2N+2)a_{319} - a_{320}],$$
$$B_{311} = t_3 A_{311} - t_4[(3N-1)a_{321} - a_{322}],$$
$$B_{312} = t_3 A_{312} - t_4[\quad 3N a_{323} - a_{324}],$$
$$B_{313} = t_3 A_{313},$$
$$a_{31} = A_{11}A_{21} + \mu B_{11}B_{21},$$
$$a_{32} = A_{11}B_{21} + B_{11}A_{21} + \mu(B_{11}A_{21} + A_{11}B_{21}) - 4Nc_A\mu\{B_{21} - \mu B_{11} + c_A\mu^2,$$
$$a_{33} = A_{11}A_{22} + A_{12}A_{21} + \mu(B_{11}B_{22} + B_{12}B_{21}),$$
$$\begin{aligned} a_{34} = &(A_{11}B_{22} + 2A_{12}B_{21} + 2B_{11}A_{22} + B_{12}A_{21}) \\ &+\mu(B_{11}A_{22} + 2B_{12}A_{21} + 2A_{11}B_{22} + A_{12}B_{21}) \\ &-4N\{c_A A_{21} + A_{11}B_{11} + c_A\mu(B_{22} - B_{21})\} \\ &+4N\mu\{A_{11} + \mu B_{12} - (4N-1)B_{11}\} + 4Nc_A^2\mu^2(6N-1), \end{aligned}$$
$$a_{35} = A_{11}A_{23} + A_{12}A_{22} + \mu(B_{11}B_{23} + B_{12}B_{22}),$$
$$\begin{aligned} a_{36} = &A_{11}B_{23} + 2A_{12}B_{22} + 3B_{11}A_{23} + 2B_{12}A_{22} \\ &+\mu\{B_{11}A_{23} + 2B_{12}A_{22} + 3A_{11}B_{23} + 2A_{12}B_{22}\} \\ &-4N\{c_A A_{22} + A_{11}B_{12} + A_{12}B_{11} + c_A\mu(B_{23} - B_{22})\} \\ &+4Nc_A\{\mu A_{12} - 2NA_{11} - \mu(4N-1)B_{12} + 2N\mu B_{11}\} - 16N^2c_A^2\mu(3N-1), \end{aligned}$$
$$a_{37} = A_{12}A_{23} + \mu B_{12}B_{23},$$
$$\begin{aligned} a_{38} = &2A_{12}B_{23} + 3B_{12}A_{23} + \mu(2B_{12}A_{23} + 3A_{12}B_{23}) \\ &-4N\{A_{12}B_{12} + c_A A_{23} - c_A\mu B_{23}\} - 8N^2c_A\{A_{12} - \mu B_{12}\} + 16N^3c_A^2\mu, \end{aligned}$$
$$a_{39} = A_{11}A_{24} + A_{13}A_{21} + \mu(B_{11}B_{24} + B_{13}B_{21}),$$
$$\begin{aligned} a_{310} = &A_{11}B_{24} + NA_{13}B_{21} + NB_{11}A_{24} + B_{13}A_{21} + \mu \\ &[B_{11}A_{24} + NB_{13}A_{21} + NA_{11}B_{24} + A_{13}B_{21} - 4Nc_A\{B_{24} - \mu B_{13}\}], \end{aligned}$$
$$a_{311} = A_{11}A_{25} + A_{12}A_{24} + A_{13}A_{22} + \mu(B_{11}B_{25} + B_{12}B_{24} + B_{13}B_{22}),$$
$$\begin{aligned} a_{312} = &A_{11}B_{25} + NA_{13}B_{22} + 2A_{12}B_{24} + (N+1)B_{11}A_{25} + NB_{12}A_{24} + 2B_{13}A \\ &+\mu\{B_{11}A_{25} + NB_{13}A_{22} + 2B_{12}A_{24} + (N+1)A_{11}B_{25} + NA_{12}B_{24} \\ &+2A_{13}B_{22}\} - 4N\{c_A A_{24} + A_{11}B_{13} + A_{13}B_{11} + c_A\mu(B_{25} - B_{24})\} \\ &+4Nc_A\mu\{A_{13} - (4N-1)B_{13}\}, \end{aligned}$$
$$a_{313} = A_{12}A_{25} + A_{13}A_{23} + \mu(B_{12}B_{25} + B_{13}B_{23}),$$
$$\begin{aligned} a_{314} = &2A_{12}B_{25} + NA_{13}B_{23} + (N+1)B_{12}A_{25} + 3B_{13}A_{23} \\ &+\mu\{2B_{12}A_{25} + NB_{13}A_{23} + (N+1)A_{12}B_{25} + 3A_{13}B_{23}\} \end{aligned}$$

$$-4N\{c_A A_{25} + A_{12}B_{13} - A_{13}B_{12} - c_A\mu B_{25}\} - 8N^2 c_A\{A_{13} - \mu B_{13}\},$$

$$a_{315} = A_{11}A_{26} + A_{13}A_{24} + \mu(B_{11}B_{26} + B_{13}B_{24}),$$

$$a_{316} = A_{11}B_{26} + NA_{13}B_{24} + \mu B_{11}A_{26} + NB_{13}A_{24} + \mu$$

$$\{B_{11}A_{26} + NB_{13}A_{24} + \mu A_{11}B_{26} + NA_{13}B_{24}\} - 4Nc_A\mu B_{26},$$

$$a_{317} = A_{11}A_{27} + A_{12}A_{26} + A_{13}A_{25} + \mu(B_{11}B_{27} + B_{12}B_{26} + B_{13}B_{25})$$

$$a_{318} = A_{11}B_{27} + 2A_{12}B_{26} + NA_{13}B_{25} + 2NB_{11}A_{27} + \mu B_{12}A_{26}$$

$$+(N+1)B_{13}A_{25} + \mu\{B_{11}A_{27} + 2B_{12}A_{26} + NB_{13}A_{25} + 2NA_{11}B_{27}$$

$$+\mu A_{12}B_{26} + (N+1)A_{13}B_{25}\} - 4N\{c_A A_{26} + A_{13}B_{13} + c_A\mu(B_{27} - B_{26})\},$$

$$a_{319} = A_{11}A_{27} + \mu B_{12}B_{27},$$

$$a_{320} = 2A_{12}B_{27} + 2NB_{12}A_{27} + \mu(2B_{12}A_{27} + 2NA_{12}B_{27}) - 4Nc_A\{A_{27} - \mu B_{27}\},$$

$$a_{321} = A_{13}A_{26} + \mu B_{13}B_{26},$$

$$a_{322} = NA_{13}B_{26} + \mu B_{13}A_{26} + \mu\{NB_{13}A_{26} + \mu A_{13}B_{26}\},$$

$$a_{323} = A_{13}A_{27} + \mu B_{13}B_{27},$$

$$a_{324} = N\{(4N-1)A_{13}B_{27} + (\mu+2)B_{13}A_{27}\},$$

$$t_3 = (5N-1)/[\mu(3N+1)],$$

$$t_4 = -1/[2c_A\mu(3N+1)].$$

5.0 Solutions in region C ($q_T \geq q > q_I$)

5.1 Zero order solutions:

From equation (9) we get, s_o = constant. By the boundary conditions (21) we get, $s_o = s_A$. In region C and D the boundary conditons require that we must choose constant zero-order solutions in C. Therefore we get the following zero-order solutions in region C,

$$u_o = A_{oo} + A_{o1}q_{T_o}, \qquad c_o = B_{oo} + B_{o1}q_{T_o}, \qquad s_o = C_{oo} + C_{o1}q_{T_o}. \tag{33}$$

5.2 First order solutions :

Solving (10) to (12) after substituting first order solutions and their derivatives we get first order solutions as

$$u_1 = g_{10} + g_{11}q, \qquad c_1 = h_{10} + h_{11}q, \qquad s_1 = i_{10} + i_{11}q, \tag{34}$$

where

$$g_{10} = q_{T_o}[(N-1)k_1 + Nk_2]/2,$$
$$g_{11} = -N[k_1 + k_2]/2,$$
$$h_{10} = q_{T_o}[(N-1)k_1 - Nk_2]/2\mu - 2c_A q_{T_o}(1 - q_{T_o}),$$
$$h_{11} = -N(k_1 - k_2)/[2\mu],$$
$$i_{10} = \mu q_{T_o} k_3,$$
$$i_{11} = -2Nk_3,$$

The solutions are valid for $N \neq 0, \frac{1}{2}$.

k_1, k_2, k_3 are integrating constants.

5.3 Second order solutions

Substituting n=2, zero-order solutions and their derivatives in (13) to (15), an solving we get second order solutions as

$$u_2 = g_{20} + g_{21}q + g_{22}q^2, \qquad c_2 = h_{20} + h_{21}q + h_{22}q^2, \qquad s_2 = i_{20} + i_{21}q + i_{22}q^2, \quad (3$$

where

$$g_{20} = [(N-1)^2 q_{T_o}^2 k_4 + \{\mu g_6 + g_8\} q_{T_o}/4N^2 + N^2 q_{T_o}^2 k_5 - g_5/2N]/2,$$
$$g_{21} = -[2N(N-1)q_{T_o}k_4 + 2N^2 q_{T_o} k_5 + g_6/N]/2,$$
$$g_{22} = N^2[k_4 + k_5]/2,$$
$$h_{20} = [(N-1)^2 q_{T_o}^2 k_4 + \{\mu g_8 - g_6\} q_{T_o}/4N^2 - N^2 q_{T_o}^2 k_5 - g_7/2N]/[2\mu],$$
$$h_{21} = -[2N(N-1)q_{T_o}k_4 - 2N^2 q_{T_o} k_5 + g_8/N]/[2\mu],$$
$$h_{22} = N^2[k_4 - k_5]/[2\mu],$$
$$i_{20} = \mu^2 q_{T_o}^2 k_6,$$
$$i_{21} = -4N\mu q_{T_o} k_6,$$
$$i_{22} = 4N^2 k_6,$$
$$g_5 = -[g_{10}g_{11} + \mu h_{10} h_{11}]/c_A,$$
$$g_6 = -[g_{11}^2 + \mu h_{11}^2]/c_A,$$
$$g_7 = 4Nq_{T_o}g_{10} + 4N\mu(1 - q_{T_o})h_{10}, -[g_{11}h_{10} + \mu g_{10}h_{11}]/c_A$$
$$-4Nc_A\mu^2 q_{T_o}(1 - q_{T_o}),$$
$$g_8 = 4Nq_{T_o}g_{11} + 4N\mu(1 - q_{T_o})h_{11} - 2Ng_{11}h_{11}/c_A + 8N^2 c_A \mu q_{T_o}(1 - q_{T_o}),$$

$$G_1(q) = -[u_1' u_1 + \mu c_1' c_1]/c_A,$$
$$G_2(q) = -[\mu c_1' u_1 + u_1' c_1 - 4N(c_o u_1 + u_o c_1 - \lambda u_o c_o)]/c_A,$$

where k_4, k_5, k_6 are integrating constants.

5.4 Third order solutions

Substituting n=3, the zero-order solutions and their derivatives in (13) to (15) and solving them analytically we get for $N \neq 0,\ \frac{1}{2}$

$$
\begin{aligned}
u_3 &= g_{30} + g_{31} q + g_{32} q^2 + g_{33} q^3, \\
c_3 &= h_{30} + h_{31} q + h_{32} q^2 + h_{33} q^3, \\
s_3 &= i_{30} + i_{31} q + i_{32} q^2 + i_{33} q^3,
\end{aligned}
\tag{36}
$$

where

$$
\begin{aligned}
g_{30} &= [q_{T_s}^3\{(N-1)^3 k_7 + N^3 k_8\} - q_{T_s}^2\{(2N^2 - \mu + 2)G_3'' - \mu G_4''\}/12N^3 \\
&\quad + q_{T_s}\{\mu g_{51} - g_{61}\}/12N^2 - g_{50}/3N]/2, \\
g_{31} &= [-3N q_{T_s}^2\{(N-1)^2 k_7 + N^2 k_8\} + q_{T_s}\{\mu G_3'' - G_4''\}/6N^2 \\
&\quad + q_{T_s}\{\mu g_{52} - g_{62}\}/6N^2 - g_{51}/2N], \\
g_{32} &= [3N^2 q_{T_s}\{(N-1)k_7 + N k_8]/2 - G_3''/6N - 2g_{52}/3N\}, \\
g_{33} &= -N^3(k_7 + k_8)/2, \\
h_{30} &= [q_{T_s}^3\{(N-1)^3 k_7 - N^3 k_8\} + q_{T_s}^2\{\mu G_3'' - (2N^2 - \mu + 2)G_4''\}/12N^3 \\
&\quad - q_{T_s}\{g_{51} - \mu g_{61}\}/12N^2 - g_{60}/3N]/[2\mu], \\
h_{31} &= [-3N q_{T_s}^2\{(N-1)^2 k_7 - N^2 k_8\} - q_{T_s}\{G_3'' - \mu G_4''\}/6N^2 \\
&\quad - q_{T_s}\{g_{52} - \mu g_{62}\}/6N^2 - g_{61}/2N]/[2(2N-1)], \\
h_{32} &= [3N^2 q_{T_s}\{(N-1)k_7 - N k_8\} - G_4''/6N - 2g_{62}/3N]/[2\mu], \\
h_{33} &= -N^3(k_7 - k_8)/[2\mu], \\
i_{30} &= \mu^3 q_{T_s}^3 k_9, \\
i_{31} &= -6N\mu^2 q_{T_s}^2 k_9, \\
i_{32} &= 12N^2 \mu q_{T_s} k_9, \\
i_{33} &= -8N^3 k_9.
\end{aligned}
$$

$$
\begin{aligned}
g_{50} &= -[g_{11} g_{20} + g_{10} g_{21} + \mu(h_{11} h_{20} + h_{10} h_{21})]/c_A, \\
g_{51} &= -2[g_{11} g_{21} + g_{10} g_{22} + \mu(h_{11} h_{21} + h_{10} h_{22})]/c_A, \\
g_{52} &= -3[g_{11} g_{22} + \mu h_{11} h_{22}]/c_A, \\
g_{60} &= -[\mu(h_{11} g_{20} + h_{21} g_{10}) + g_{11} h_{20} + g_{20} h_{10} \\
&\quad - 4N\{c_o g_{20} + g_{10} h_{10} + u_o h_{20} - \mu(c_o g_{10} + u_o h_{10}) - \mu^2 c_o u_o\}]/c_A,
\end{aligned}
$$

$$p_{61} = -[\mu(h_{11}g_{21} + h_{21}g_{11} + 2h_{22}g_{10}) + g_{11}h_{21} + g_{21}h_{11} + 2g_{20}h_{10}$$
$$-4N\{c_o g_{21} + g_{10}h_{11} + 2g_{20}h_{10} + u_o h_{21}$$
$$-\mu(c_o g_{11} + u_o h_{11}) + 2N(c_o g_{10} + u_o h_{10}) - 4N\mu c_o u_o\}]/c_A,$$
$$p_{62} = -[\mu(h_{11}g_{22} + 2h_{22}g_{11}) + g_{11}h_{22} + 2g_{22}h_{11}$$
$$-4N\{c_o g_{22} + g_{11}h_{11} + u_o h_{22} + 2N(c_o g_{11} + u_o h_{11}) - 4N^2 c_o u_o\}]/c_A,$$
$$G_3(q) = -(u_1' u_2 + u_2' u_1) - \mu(c_1' c_2 + c_2' c_1)$$
$$G_4(q) = -\mu(c_1' u_2 + c_2' u_1) - u_1' c_2 - u_2' c_1$$
$$+4N\{c_o u_2 + c_1 u_1 + c_2 u_o - (c_o u_1 + u_o c_1)\lambda + u_o c_o \lambda^2\}.$$

k_7, k_8, k_9 are integrating constants.

6.0 Solutions in region D $(q_I \geq q > q_S)$

6.1 Zero order solutions

In this region the zero-order solution are obtained by taking the boundary condi

tion at the interface, we get (3

$$u_o = A_o, \qquad c_o = B_o, \qquad s_o = C_o,$$

where A_o, B_o, C_o are constants to be determined by matching the conditions (24) and

given by

$$A_o = c_A \mu (1 - q_{T_o}),$$

$$B_0^{\mu+2} e^{-rC_o} = (c_A q_{T_o})^{\mu+2} e^{-rs_A}.$$

6.2 First order solutions

The solutions are similar to those in region C. Substituting zero order sol

and their derivatives in (10) to (12), and solving them analytically we get the first

solutions as

$$u_1 = G_{10} + G_{11} q, \qquad c_1 = H_{10} + H_{11} q, \qquad s_1 = I_{10} + I_{11} q,$$

where

$$G_{10} = K_1[2c_A N q_{I_o} - B_o]/2 + K_2[2c_A N q_{I_o} + B_o]/2 - B_o^2 K_3/[2c_A N],$$
$$G_{11} = -c_A N[K_1 + K_2]/2,$$
$$H_{10} = [K_1(2c_A N q_{I_o} - B_o) - K_2(2c_A N q_{I_o} + B_o)]/2\mu - 2A_o B_o/c_A \mu,$$
$$H_{11} = -c_A N[K_1 - K_2]/\mu,$$
$$I_{10} = -K_3 q_{I_o},$$
$$I_{11} = K_3,$$

where K_1, K_2, K_3 are integrating constants. These solutions are valid for $N \neq$

6.3 Second order solutions

Substituting n=2, zero-order solutions and their derivatives in (13) to (15) and solving we get the second order solutions as,

$$u_2 = G_{20} + G_{21} q + G_{22} q^2, \qquad c_2 = H_{20} + H_{21} q + H_{22} q^2, \qquad s_2 = I_{20} + I_{21} q + I_{22} q^2, \quad (39)$$

where

$$\begin{aligned}
G_{20} &= [(2c_A N q_{I_o} - B_o)^2 K_4 + (2c_A N q_{I_o} + B_o)^2 K_5 + H_1' q_{I_o}/2c_A N \\
&\quad - H_2' B_o/4c_A^2 N^2 - \{B_o^2 I_{21} - u_1' G_{10} + 2B_o s_1' H_{10} - \mu c_1' H_{10}\}/2c_A N]/2, \\
G_{21} &= -[H_1' + 2B_o^2 I_{22} - u_1' G_{11} + 2B_o s_1' H_{11} - \mu c_1' H_{11}]/4c_A N \\
&\quad - 2N c_A (2c_A N q_{I_o} - B_o) K_4 - 2N c_A (2c_A N q_{I_o} + B_o) K_5, \\
G_{22} &= 2c_A^2 N^2 (K_4 + K_5), \\
H_{20} &= [(2c_A N q_{I_o} - B_o)^2 K_4 - (2c_A N q_{I_o} + B_o)^2 K_5 - H_1' B_o/4c_A^2 N^2 + H_2' q_{I_o}/2c_A N \\
&\quad - \{(4NB_o - \mu c_1') G_{10} + (4NA_o - u_1') H_{10} - 4NA_o B_o \mu\}/2c_A N]/2\mu, \\
H_{21} &= -[H_2' + \{4NB_o - \mu c_1'\} G_{11} + (4NA_o - u_1') H_{11} + 8A_o B_o N^2]/ \\
&\quad [\mu 4 c_A N] - 2N c_A [(2c_A N q_{I_o} - B_o) K_4 - (2c_A N q_{I_o} + B_o) K_5]/\mu, \\
H_{22} &= 2c_A^2 N^2 (K_4 - K_5)/\mu, \\
I_{20} &= 4N^2 c_A^2 q_{I_o}^2 K_6 + K_3 (K_1 + K_2) q_{I_o}/4 + K_3 G_{10}/4N c_A, \\
I_{21} &= -8N^2 c_A^2 q_{I_o} K_6 - K_3 (K_1 + K_2)/4 + K_3 G_{11}/4N c_A, \\
I_{22} &= 4N^2 c_A^2 K_6, \\
H_1(q) &= B_o^2 s_2' - u_1' u_1 + [2B_o s_1' - \mu c_1'] c_1, \\
H_2(q) &= [4NB_o - \mu c_1'] u_1 + (4NA_o - u_1') c_1 - 4\lambda N A_o B_o.
\end{aligned}$$

ere K_4, K_5, K_6 are integrating constants.

6.4 Third order solutions

Substituting n=3, the zero order solutions and their derivatives in (13) and (15), and solving the resulting equations analytically, we get third order solutions as

$$
\begin{aligned}
u_3 &= G_{30} + G_{31}q + G_{32}q^2 + G_{33}q^3, \\
c_3 &= H_{30} + H_{31}q + H_{32}q^2 + H_{33}q^3, \\
s_3 &= I_{30} + I_{31}q + I_{32}q^2 + I_{33}q^3,
\end{aligned}
\qquad (40)
$$

where

$$
\begin{aligned}
G_{30} = {} & \{(2Nc_A q_{I_0} - B_0)^3 K_7 + (2Nc_A q_{I_0} + B_0)^3 K_8 - H_5''(4N^2c_A^2 q_{I_0}^2 + B_0^2)/24N^3c_A^3 \\
& + H_4'' q_{I_0} B_0/6N^2c_A^2 + (2Nc_A q_{I_0} G_{51} - B_0 G_{61})/12N^2c_A^2 - G_{50}/3Nc_A\}/2, \\
G_{31} = {} & \{-6Nc_A(2Nc_A q_{I_0} - B_0)^2 K_7 - 6Nc_A(2Nc_A q_{I_0} + B_0)^2 K_8, \\
& + (2Nc_A q_{I_0} H_5'' - B_0 H_4'')/6N^2c_A^2 + (2q_{I_0} G_{52} - G_{51})/6Nc_A \\
& - (2Nc_A G_{51} + B_0 G_{62})/6N^2c_A^2\}/2, \\
G_{32} = {} & [12N^2c_A^2\{(2Nc_A q_{I_0} - B_0)K_7 + (2Nc_A q_{I_0} + B_0)K_8\} - (H_5'' + 4G_{52})/6Nc_A]/2, \\
G_{33} = {} & -4N^3c_A^3(K_7 + K_8), \\
H_{30} = {} & [(2Nc_A q_{I_0} - B_0)^3 K_7 - (2Nc_A q_{I_0} + B_0)^3 K_8 + H_5'' q_{I_0} B_0/6N^2c_A^2 \\
& - H_4''(4N^2c_A^2 q_{I_0}^2 + B_0^2)/24N^3c_A^3 + \{2Nc_A q_{I_0} G_{61} - B_0 G_{51}\}/12N^2c_A^2 - G_{60}/3Nc_A \\
& /[2(2n-1)] \\
H_{31} = {} & [-6Nc_A(2Nc_A q_{I_0} - B_0)^2 K_7 + 6Nc_A(2Nc_A q_{I_0} + B_0)^2 K_8 \\
& + (2Nc_A q_{I_0} H_4'' - B_0 H_5'')/6N^2c_A^2 + (2q_{I_0} G_{62} - G_{61})/6Nc_A \\
& - (2Nc_A G_{61} + B_0 G_{52})/6N^2c_A^2]/2\mu, \\
H_{32} = {} & \{12N^2c_A^2(2Nc_A q_{I_0} - B_0)K_7 - 12N^2c_A^2(2Nc_A q_{I_0} + B_0)K_8 \\
& - (H_4'' + 4G_{62})/6Nc_A\}/2\mu, \\
H_{33} = {} & -4N^3c_A(K_7 - K_8)/\mu, \\
I_{30} = {} & q_{I_0}^3 K_9 + \{2G_{70} - q_{I_0} G_{71} + 2q_{I_0}^2 G_{72}\}/12Nc_A, \\
I_{31} = {} & -3q_{I_0}^2 K_9 + \{G_{71} - 2G_{72}\}/4Nc_A, \\
I_{32} = {} & 3K_9 q_{I_0} + G_{72}/2Nc_A, \\
I_{33} = {} & -K_9, \\
G_{50} = {} & f_1 G_{20} + \{2B_0 I_{21} + s_1'\}\{H_{10} - \mu H_{21}\}H_{10} + B_0^2 I_{31} - u_1' G_{20} - G_{21}G_{10}, \\
G_{51} = {} & f_1 G_{21} + \{2B_0 I_{21} + s_1'\}\{H_{10} - \mu H_{21}\}H_{11} + \{4B_0 I_{22} + s_1'\}\{H_{11} \\
& - \mu 2H_{22}\}H_{10} + 2B_0^2 I_{32} - u_1' G_{21} - G_{21}G_{11} - 2G_{22}G_{10}, \\
G_{52} = {} & f_1 G_{22} + \{4B_0 I_{22} + s_1'\}\{H_{11} - \mu 2H_{22}\}H_{11} + 3B_0^2 I_{33} - u_1' G_{22} - 2G_{22}G \\
G_{60} = {} & -\mu(c_1' G_{20} + H_{21}G_{10}) - (u_1' H_{20} + G_{21}H_{10}) \\
& + 4N\{c_0 G_{20} + G_{10}H_{10} + u_0 H_{20} - \mu(c_0 G_{10} + u_0 H_{10}) + \mu^2 c_0 u_0\},
\end{aligned}
$$

$$G_{81} = -\mu(c_1^t G_{21} + H_{21}G_{11} + 2H_{22}G_{10}) - (u_1^t H_{21} + G_{21}H_{11} + 2G_{22}H_{10})$$
$$+4N\{(c_o G_{21} + G_{10}H_{11} + G_{11}H_{10} + u_o H_{21}) - \mu(c_o G_{11} + u_o H_{11})$$
$$+2N(c_o G_{10} + u_o H_{10}) - 4N\mu c_o u_o\},$$
$$G_{82} = -\mu(c_1^t G_{22} + 2H_{22}G_{11}) - (u_1^t H_{22} + 2G_{22}H_{11})$$
$$+4N(c_o G_{22} + G_{11}H_{11} + u_o H_{22} + 2N(c_o G_{11} + u_o H_{11}) + 4N^2 c_o u_o\},$$
$$G_{70} = G_{10}I_{21} + G_{20}I_{11},$$
$$G_{71} = G_{11}I_{21} + 2G_{10}I_{22} + G_{21}I_{11},$$
$$G_{72} = 2G_{11}I_{21} + G_{22}I_{11},$$
$$f_1 = 2B_o s_1^t - \mu c_1^t.$$
$$H_3(q) = -u_1^t u_2 - u_2^t u_1 + [2B_o s_1^t - \mu c_1^t]c_2$$
$$+[2B_o s_2^t + c_1 s_1^t - \mu c_2^t]c_1 + B_o^2 s_3^t,$$
$$H_4(q) = -\mu(c_1^t u_2 + c_2^t u_1) - (u_1^t c_2 + u_2^t c_1)$$
$$+4N\{(c_o u_2 + c_1 u_1 + c_2 u_1) - (c_o u_1 + u_o c_1)\lambda + c_o u_o \lambda^2\},$$
$$H_5(q) = s_1^t u_2 + s_2^t u_1,$$

K_7, K_8 and K_9 are integrating constants.

7.0 Determination of integrating constants

7.1 Evaluation of zero order constants

By a similar procedure given in [1] the zero order constants $A_o, B_o, C_o, q_{T_o}, q_{I_o}, q_{s_o}$ and U_o are evaluated.

7.2 First order integrating constants

Since entropy is continuous in region B, C, it follows that $s_1 = 0$, therefore

$$k_3 = 0. \tag{41}$$

Consider the linear combination $u + \mu c$. In the region C, this combination is independent of k_2. By matching the first order solutions in B and C, we get

$$[u_1 + \mu c_1]_{q_{T_o} - o} = [u_1 + \mu c_1]_{q_{T_o} + o}.$$

From the above equation we find k_1, as

$$k_1 = \frac{2c_A \mu}{(N-1)(N-2)}[(N-2) - 2(N-1)q_{T_o} + N q_{T_o}^{N-1}], \tag{42}$$

for $N \neq 1, 2$. This equation is the modified form of (5.04) of [1].

First order matching conditions for velocity and pressure at the interface are given by

$$u_1(q_{I_o} - o) = u_1(q_{I_o} + o), \tag{43}$$

$$\frac{c_1(q_{I_o}-o)}{c_A q_{T_o}} = \frac{c_1(q_{I_o}+0)}{B_o}. \tag{44}$$

(44) is the modified form of (5.10) of [1]. By matching the boundary conditions at the shock (25) to (27), and substituting $q = q_{s_o} + o$ in the first order solutions (38) of region D, one can find K_1, K_2, K_3 and k_2 in terms of U_1. The values of K_1, K_2, K_3 *and* k_2 are substituted in (43) and (44) we can find the value of U_1, which may be substituted back in equations of K_1, K_2, K_3 and k_2. The following are equations of K_1, K_2, K_3, k_2 for $N \neq 0, \frac{1}{2}, 1, 2$

$$K_1 a_o = [\frac{\mu(U_o^2+1)}{2NU_o^2} + \frac{\mu[(\mu+2)U_o^4+\mu]}{4N^2B_oU_o^3} - \frac{A_o^3}{U_o(U_o-A_o)}]U_1 + \frac{2A_oB_o}{c_A}, \tag{45}$$

$$K_2 b_o = [\frac{\mu(U_o^2+1)}{2NU_o^2} - \frac{\mu[(\mu+2)U_o^4+\mu]}{4N^2B_oU_o^3} - \frac{A_o^2}{U_o(U_o-A_o)}]U_1 - \frac{2A_oB_o}{c_A}, \tag{46}$$

$$K_3 = -\frac{2Nc_AA_o^2}{B_o^2U_o(U_o-A_o)}U_1, \tag{47}$$

$$A_1 = u_1(q_{I_o}+o) = -\frac{B_o}{2}(K_1-K_2) - \frac{B_o^2K_3}{2c_AN}, \tag{48}$$

$$k_2 = k_1 + \frac{4A_1}{q_{T_o}}, \tag{49}$$

$$c_Aq_{T_o}(K_1+K_2) + B_o(K_1-K_2) + \frac{B_o^2K_3}{c_AN} = 2c_Aq_{T_o}\mu\{2(1-q_{T_o}) - \frac{2A_o}{c_A\mu} + \frac{k_1}{2c_A\mu}\}, \tag{50}$$

where $a_o = U_o - A_o - B_o$ and $b_o = U_o - A_o + B_o$.

Substitute for $K_1, K_2, \& K_3$ in terms of U_1 in (50) and can be solved for U_1. By substituting U_1 in relation (45) to (47) we get K_1, K_2, K_3 . From (48) we get A_1 and k_2 from (49).

7.3 Second and third order integrating constants

By a similar procedure outlined in the section 7.2 for the evaluation of first order integrating constants, the second and third order integrating constants can be evaluated, by matching the respective second and third order boundary conditions at the tail, interface and the shock boundaries.

The following are the equations for the second order constats:

$$k_6 = 0. \tag{51}$$

$$q_{T_o}^2 k_4 = [u_2 + \mu c_2]_{(q_{T_o}-0)} + \frac{(g_6 + g_8)}{4N^2}(N+1)q_{T_o} + \frac{g_5 + g_7}{4N}. \tag{52}$$

$$K_4 a_o^2 = [u_2 + (2N-1)c_2]_{(q_{s_o}+0)} - \frac{H_1'(q_{s_o}) + H_2'(q_{s_o})}{8c_A^2 N^2} a_o + \frac{(H_1(q_{s_o}) + H_2(q_{s_o})}{4c_A N}, \tag{53}$$

$$K_5 b_o^2 = [u_2 - (2N-1)c_2]_{(q_{s_o}+0)} - \frac{H_1'(q_{s_o}) - H_2'(q_{s_o})}{8c_A^2 N^2} b_o + \frac{H_1(q_{s_o}) - H_2(q_{s_o}),}{4c_A N}. \tag{54}$$

$$K_6[2Nc_A(q_{I_o} - q_{s_o})]^2 = s_2(q_{s_o}+o) - \frac{K_3(K_1+K_2)}{8c_A N}[2c_A N(q_{I_o} - q_{s_o})] - \frac{K_3 u_1(q_{s_o}+o)}{4c_A N}, \tag{55}$$

$$A_2 = u_2(q_{I_o}+o) = \frac{1}{2}[B_o^2(K_4+K_5) - \frac{B_o H_2'(q_{I_o})}{4c_A^2 N^2} - \frac{H_1(q_{I_o})}{2c_A N}], \tag{56}$$

$$\frac{k_8 q_{T_o}^2}{8} = A_2 - \frac{1}{2}(\frac{k_4 q_{T_o}^2}{8} - \frac{g_8 q_{T_o}}{4N^2} - \frac{g_8 q_{I_o}}{2N} - \frac{g_5}{2N}). \tag{57}$$

$u_2(q_{I_o}+0)$, $u_2(q_{s_o}+0)$, $c_2(q_{s_o}+0)$ are values of the solutions in region D, + indicates the region to which the solution belongs.

The following are the equations for the third order constants:

$$k_9 = 0. \tag{58}$$

$$-k_7 q_{T_o}^3 = [u_3 + \mu c_3]_{q_{T_o}-0} + \left[\frac{G_3'' + G_4''}{12N^3} q_{T_o}^2 + \frac{G_4' + G_4'}{12N^2} q_{T_o} + \frac{G_3 + G_4}{6N}\right]_{q_{T_o}+0}. \tag{59}$$

$$K_7 a_o^3 = [u_3 + \mu c_3]_{(q_{s_o}+0)} + \left[\frac{H_3'' + H_4''}{48N^3 c_A^3} a_o^2 - \frac{H_3' + H_4'}{24N^2 c^2 A} a_o + \frac{H_3 + H_4}{6Nc_A}\right]_{q_{s_o}+0}, \tag{60}$$

$$K_8 b_o^3 = [u_3 - \mu c_3]_{(q_{s_o}+0)} + \left[\frac{H_3'' - H_4^{11}}{48N^3 c_A^3} b_o^2 - \frac{H_3' - H_4'}{24N^2 c_A^2} b_o + \frac{H_3 - H_4}{6Nc_A}\right]_{q_{s_o}+0}, \tag{61}$$

$$K_9[q_{I_o} - q_{s_o}]^3 = s_3(q_{s_o}+0) - \left[\frac{H_5''(q_{I_o} - q_{s_o})}{12Nc_A} - \frac{H_5'(q_{I_o} - q_{s_o})}{12Nc_A} + \frac{H_5}{6Nc_A}\right]_{q_{s_o}+0}, \tag{62}$$

$$A_3 = u_3(q_{T_o}+0) = -\frac{B_o^2}{2}(K_T - K_3) - \left[\frac{B_o^2 H_3^{11}}{48N^3 c_A^3} + \frac{B_o H_4^1}{24N^2 c_A^3} + \frac{H_3}{6N c_A}\right]_{q_{T_o}+0}, \tag{63}$$

$$k_3 \frac{q_{T_o}^4}{16} = A_3 + \left[\frac{q_{T_o}^3 k_T}{16} + \frac{q_{T_o}^2 G_3^{11}}{48N^3} + \frac{q_{T_o} G_4^1}{24N^2} + \frac{G_3}{6N}\right]_{q_{T_o}-0}. \tag{64}$$

8.0 Results and discussion

In region B, it is observed that the solutions are in the form of multinomials in q, if N is not an integer. Otherwise they are in the form of polynomials in q, except for the values of $N = 0, 1/3, , 1/2, 1, 4/3, 3/2, 2$. Generally an nth order solution consists of (n+1) degree polynomial in q, n^2 terms of multinomial in q and therefore the general form of solution is given by

$$x_n = \sum_{i=o}^{n+1} a_i q^i + \sum_{k=o}^{n-1} q^{(k+1)N} \left(\sum_{j=o}^{n-1} b_{k,j} q^{j-k} \right),$$

where x_n can be particle velocity, sound speed or entropy.

In region C and D the solutions are polynomials in q except for N=0, 1/2. The nth order solution x_n can be represented as

$$x_n = \sum_{i=o}^{n} a_i q^i$$

The mathematical model developed has been computerised and run for the results. In table 1 zero, first, second, and third order solutions of the particle velocity is tabulated. The particle velocity increases upto the boundary of regions B and C and then starts de- creasing, except in the zero order solutions. In the case of zero order solutions the velocity increases upto the boundary of regions B and C and then remains constant in regions C and D. It can be observed that the particle velocity of every order is continuous at the boudaries of regions B and C, C and D and the solutions numerically converge upto the fourth decima place.

In table 2, the pressure parameter is tabulated. The pressure decreases right fro the boundary of regions A and B without any jump at the boundaries of B, C and D. At t shock front the value of pressure is 4.2525 (3rd order solution). The convergence of pressu can be seen upto third place.

In Fig 2 and 3, the particle velocity, and pressure are plotted against radius are compared with that of the results obtained from Random Choice Method(RCM) [6

9.0 Conclusions

McFadden [1] obtained the solutions upto first order and has given pressure curve against the radius for a particular case for N=3 and γ =1.4. But the boundary condition for first order sound velocity given at eqn (5.10) and solutions (4.13) to 4.15 of [1] in region C are not satisfying the first order equations of motion (4.10) to (4.12). Also there is discrepancy in the equation (5.04) given for the evaluation of k_1 and the modified equation is given at eqn (42). The modified form of (4.13) to (4.15) is given at equation (34). Due to these discrepancies the equation (6.01) to (6.06) of [1] are to be modified. The generalised form of these equations for any value of N and γ are given at (45) to (50). The pressure obtained are compared with that of results of RCM [6].

10.0 Acknowledgments

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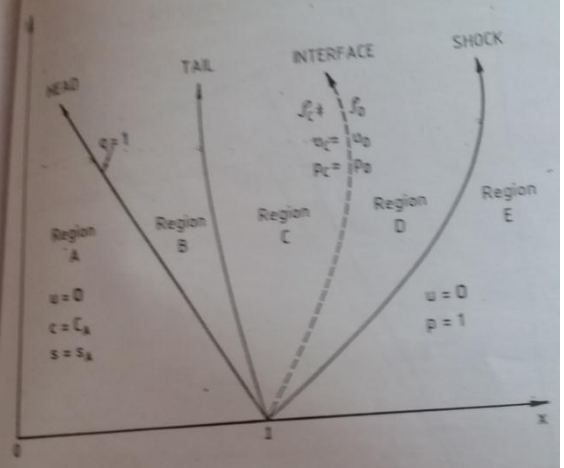

Fig. 1: SPACE-TIME DIAGRAM FOR A SPHERICAL BLAST

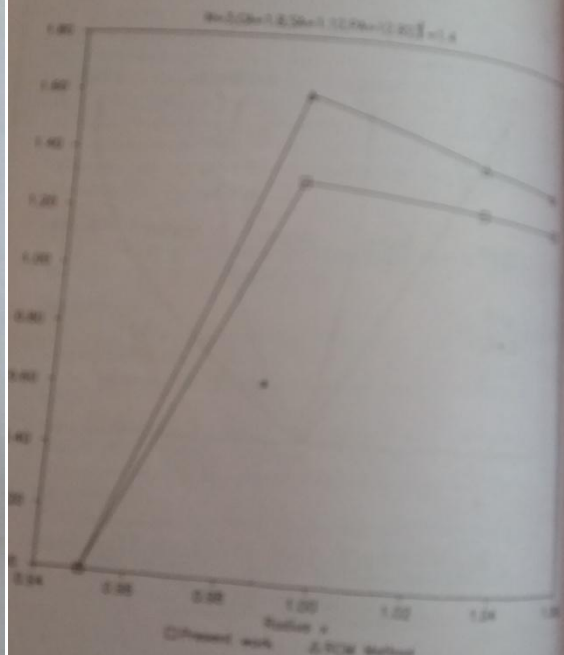

Fig 2. Particle velocity profile vs. radius in spherical blasts

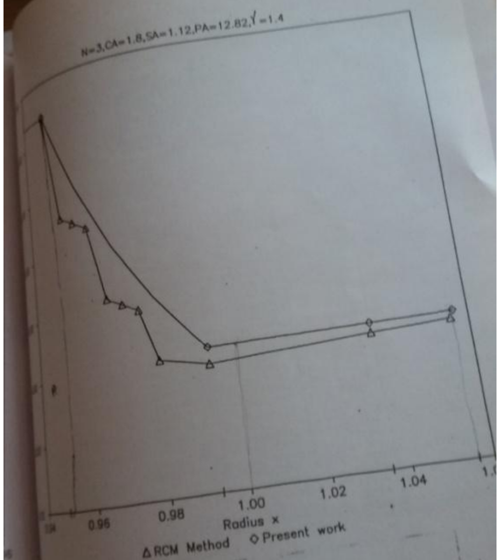

Fig 3. Pressure distribution in spherical blasts