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A Model for Estimation of Aircraft Attrition from Various Ground Air Defence Weapons

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ABSTRACT

A computer-based mathematical model is developed for the estimation of assessment of damage inflicted on an aircraft due to a ground-based air defence gun. It is assumed that the aircraft is approaching the target from an arbitrary direction and does not change it's trajectory during gun firing. Dimension of aircraft and trajectory of warhead are assumed to be known. Damage to aircraft is caused due to blast as well as fragments. Aircraft is assumed to be killed if one of its vital parts has been killed.

1. INTRODUCTION

Air defence (AD) guns and missiles are deployed to provide protection against hostile aircraft coming to attack vulnerable areas and vulnerable points. These weapons may be single- or multi-barrel and may fire DA- or VT-fuzed ammunition or warheads. In order to identify a suitable AD weapon for purposes such as acquisition, or design and development or deployment, so that it is desirable to make an assessment of its effectiveness. The problem of assessing the effectiveness of AD weapons to stationary as well as mobile targets has been studied by various authors[1-5]. While the aircraft has been modelled as a right cylinder and presenting a circular target of some dimensions by the earlier authors, here we have considered the aircraft comprising of various sections modelled as cones, cylinders, wedges, etc. Further, the aircraft is

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not considered to be necessarily radially approaching the target, which has been the assumption in most of the earlier works. In the present report, we have discussed damage to aircraft body due to explosive charge as well as due to fragments, when warhead/ammunition explodes in the near vicinity of the aircraft. Kill criterion has been taken as the minimum number of fragments required to penetrate and kill a particular part. In the case of blast waves, it is assumed that the probability of kill is one, based upon the impluse transmitted to the structure. A typical aircraft and a typical AD gun with DA/VT-fuzed ammunition has been considered for the validation of the model. However, the model is quite general and can be used for all types of aircraft/weapons.

The aim of the paper is to develop a computer-based mathematical model for the assessment of damage inflicted on an aircraft, using an AD weapon. It is assumed that the dimensions and orientation of the aircraft and the shell/warhead are known.

2. MATHEMATICAL MODEL

An aircraft can be considered to be divided into a number of parts some of which are vital parts such as cockpit, engine, fuel tank, control unit, etc. Aircraft can be considered as killed if at least one of these vital components is killed. Damage to aircraft is caused by the blast when explosion is in its near vicinity and by fragments, if it is at a distance. In the present report we have studied damage due to blast as well as fragmentation effect. A DA-fuzed ammunition defeats the target by first making a physical impact and then exploding. While the penetration is governed by the kinetic energy of the projectile at the point of impact, the structural damage is decided by the pressure transmitted to the aircraft body due to the explosion of the charge. In the case of VT-fused ammunition, it first reaches in the vicinity of the target and explodes at a predetermined distance. Fragments thus formed, hit at various parts of the target and cause damage. Kill at the target aircraft is based on the kill of its vital parts. A vital part is assumed to be killed if required amount of energy is transmitted to the part by the fragments.

In the following section, models for DA- as well as VT-fused ammunition have been developed.

3. MODEL FOR DA-FUZED AMMUNITION

The probability of kill of an aircraft depends on various functions such as kill of its vital parts, probability of hit, probability of fuze-functioning, etc. It is not necessary that even if a part of aircraft is damaged fully, aircraft is killed. It is known from war experiences, that quite a number of aircraft return to friendly areas even after being damaged heavily. Probability of kill P_k of an aircraft component can be defined as

$$P_k = P_h \cdot P_f \, P_{k/hf} \tag{1}$$

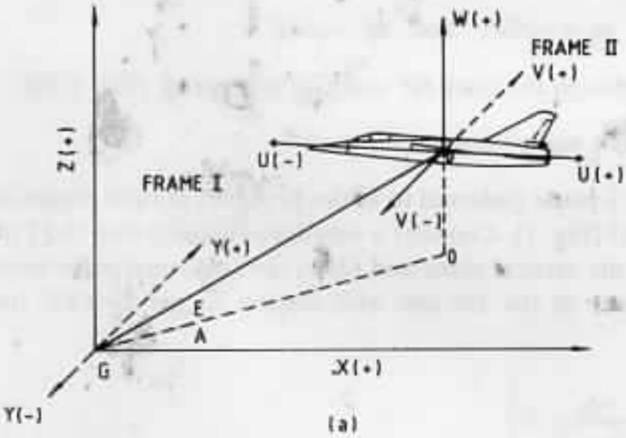
where P_h is the single shot hit probability of the ammunition at the component; P_f is given that the ammunition has hit the component and the fuze has functioned.

Evaluation of these three probabilities will be discussed in the following sections.

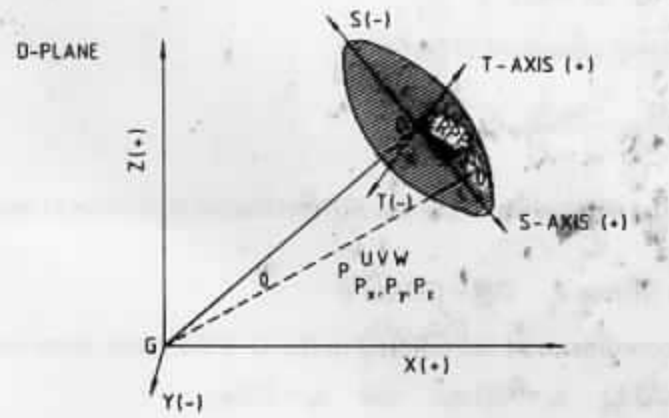
3.1 Single Shot Hit Probability

For the purpose of finding single shot hit probability (SSHP) of a round of ammunition, we consider an earthfixed rectangular frame of reference *G-XYZ* in which the origin *G* is at the weapon position and the axes of the frame *G-XYZ* (Frame-I) are orthogonal and a moving orthogonal frame of reference *O-UVW* (Frame-II) in which the origin *O* is at the geometrical centre of the aircraft and the axes *OU*, *OV*, *OW* respectively are along the rolling, pitching and yawing axis of the aircraft (Fig. 1). Then, direction cosines (l_0, m_0, n_0) of the line *GO* are given by

$$l_0 = \cos A \cos E; \quad m_0 = \sin A \cos E; \quad \text{and} \quad n_0 = \sin E \tag{2}$$



(a)



(b)

where A and E are respectively the angles in azimuth and elevation of the aircraft (Fig. 1). If (x,y,z) and (u,v,w) respectively are the coordinates of a point on the aircraft with respect to fixed and moving frame of reference, then the relations between the coordinates (x_p, y_p, z_p) and (u_p, v_p, w_p) of a point P at the aircraft are easily seen to be as follows :

$$x_p = x_0 + l_1 \cdot u_p + l_2 \cdot v_p + l_3 \cdot w_p$$
$$y_p = y_0 + m_1 \cdot u_p + m_2 \cdot v_p + m_3 \cdot w_p$$
$$z_p = z_0 + n_1 \cdot u_p + n_2 \cdot v_p + n_3 \cdot w_p \tag{3}$$

where (x_0, y_0, z_0) are the coordinates of the aircraft centre with respect to the G-XYZ frame and (l_i, m_i, n_i), $i = 1, 2, 3$ are respectively the direction cosines of OU, OV, OW with respect to fixed frame of reference. Now direction cosines of line GP are given by

$$l_p = x_p/GP; \quad m_p = y_p/GP; \quad \text{and} \quad n_p = z_p/GP \tag{4}$$

and the angle θ between the lines GP and GO is given by (Fig. 1(b))

$$\theta = \cos^{-1}(l_0 l_p + m_0 m_p + n_0 n_p) \tag{5}$$

Now consider a plane (referred to as the D-plane) at right angles to GO passing through the point O (Fig. 1). Consider a two-dimensional frame O-ST in the D-plane such that OT is in the vertical plane and OS in the horizontal plane through O. Then the direction cosines of the OS-axis with respect to the G-XYZ frame are (see Appendix)

$$\left(\frac{m_0}{\sqrt{1-n_0^2}}, \frac{-l_0}{\sqrt{1-n_0^2}}, 0\right)$$

and the direction cosines of OT-axis are

$$\left(\frac{-l_0 n_0}{\sqrt{1-n_0^2}}, \frac{-m_0 n_0}{\sqrt{1-n_0^2}}, \sqrt{1-n_0^2}\right)$$

If Q be the point in which the line GP (produced, if necessary) meets the D-plane, then

$$GQ = GO/\cos\theta, \quad OQ = GO \tan\theta \tag{6}$$

Therefore, coordinates of the point Q in the G-XYZ frame turns out to be

$$x_q = GQ \cdot l_p; \quad y_q = GQ \cdot m_p; \quad \text{and} \quad z_q = GQ \cdot n_p \tag{7}$$

Also, the direction cosines of the line OQ with respect to the G-XYZ frame are

Finally, the coordinates (s_q, t_q) of the point Q in the D-plane are given by

$$s_q = OQ \cdot \cos\phi; \quad \text{and} \quad tq = OQ \cdot \cos\psi \tag{9}$$

where

$$\cos\phi = l_q \cdot l_s + m_q \cdot m_s + n_q \cdot n_s; \quad \text{and} \quad \cos\psi = l_q \cdot l_t + m_q \cdot m_t + n_q \cdot n_t$$

(l_s, m_s, n_s) being the direction cosines of the S-axis with respect to the G-XYZ frame and (l_t, m_t, n_t) are the direction cosines of the T-axis with respect to the G-XYZ frame.

Let F_p be the shape of a typical part of the aircraft body bounded by line segments with vertices P_i $(i=1,2,....,n)$, then corresponding points Q_i $(i=1,2,....n)$ of the projection of the part on D-plane can be determined as explained above, and a corresponding figure F_q can be obtained. The figure F_q is such that a hit on this will imply a hit on the figure F_p of the aircraft body. Similar analogy can be extended for other parts of the aircraft even those parts, which are bounded by curved segments.

Finally, if σ_s and σ_t be the standard deviations of the normal distribution governing the points of impact of the rounds on the aircraft, then SSHP on a figure F_p of the aircraft is given by

$$P_h = \frac{1}{2\pi\sigma_s\sigma_t} \iint_{F_q} \exp\left\{-1/2\left(\frac{s^2}{\sigma_s^2} + \frac{t^2}{\sigma_t^2}\right)\right\} ds\, dt \tag{10}$$

It is assumed that the round has been aimed at the centre of the aircraft. The parameters σ_s and σ_t can be computed from the system errors of the weapon in the azimuth and elevation respectively.

3.2 Probability of Fuze-Functioning

The probability of fuze-functioning P_f for a DA-fuzed ammunition is constant and a part of the data, and has been taken as 1.0 in the present case.

3.3 Probability of Kill

The probability of kill in one round of DA charge may be taken as 1.0 as the explosive energy released by the shell is much higher than the energy required by any of vital components of the aircraft to kill it[6].

4. MODEL FOR VT-FUZED AMMUNITION

The VT-fuzed ammunition shell first reaches in the vicinity region (Fig. 2) of the target aircraft then its fuze functions and explodes into fragments having high kinetic energy by vicinity region, it is meant, the region around the aircraft's structure in

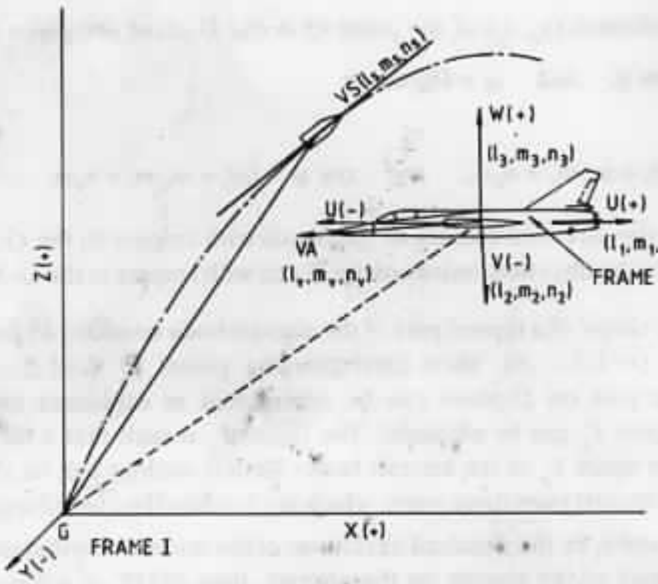


Figure 2. The position of aircraft and the VT-shell when the shell is likely to burst at a point CS in the vicinity region of the aircraft (all parameters are referred w.r.t. Frame-I).

The probability of kill of a component of the aircraft in one round can be given as

$$P_k = \int_0^{RL} PLD_c(r) \cdot Pdf(r) dr + \int_{RL}^{RU} PLD(r) \cdot Pdf(r) \cdot P_{km}(r) dr \tag{11}$$

where RL is the distance from the surface of the aircraft within which, if the VT shell explodes the shock wave itself can damage the aircraft's component; RU is the vicinity limit, a distance from the surface of the aircraft beyond which the shell cannot explode Vicinity shell is a shell formed by an imaginery surface at a distance r from the surface of the aircraft around it . $PLD_c(r)$ is the probability that the VT shell will land around the component at a distance r; $Pdf(r)$ is the probability that the fuze will function at a distance r from the aircraft's surface; $PLD(r)$ is the probability that VT shell will land around the aircraft at a distance r from the surface of the aircraft; $P_{km}(r)$ is the probability that at least k number of fragments of mass $\geq m$ will penetrate the component's structure; and k is the number of lethal fragment hits required to kill the component.

4.1 Determination of *RL*

The estimation of RL can be done on the basis of critical impulse failure criterion[7].

$$I_c = (\rho/E)^{1/2} \cdot t \cdot \sigma_y \tag{12}$$

where E is Young's modulus, ρ is density of material, t is thickness, and σ_y is dynamic yield strength.

In applying this method to skin panels supported by transverse longitudinal members, for example, one first calculates the critical impluse and natural period of the panel. Incident pressure pulse having a duration of one-quarter of the natural period or more having an impulse at least equal to I_c will cause rupture of the panel at the attachments.

If the distance of point of explosion from the target is z, then

$$p^0/p_a = \frac{808[1 + (z/4.5)^2]}{\sqrt{1 + (z/0.048)^2}\,\sqrt{1 + (z/0.32)^2}\,\sqrt{1 + (z/1.35)^2}} \tag{13}$$

where p^0 is the incident blast wave, and p_a is the atmospheric pressure. Time duration t_d of positive phase of shock is given by the following relation,

$$\frac{t_d}{w^{1/3}} = \frac{980[1 + (z/0.54)^{10}]}{[1 + (z/0.02)^3]\,[1 + (z/0.74)^6]\,\sqrt{[1 + (z/6.9)^2]}} \tag{14}$$

where w is the charge weight in kg. Reflected pressure p_r can be given as

$$p_r = \frac{p_a(8p^0/p_a + 7)\,(p^0/p_a + 1)}{(p^0/p_a + 7)} \tag{15}$$

Therefore the total impulse I can be given by

$$I = \int_0^{t_d} P_r dt \tag{16}$$

If the reflected pressure pulse has been assumed to be a triangular pressure pulse, then

$$I = \frac{P_r \cdot t_d}{2} \tag{17}$$

Taking the dimensions of the panel as a and b, the natural frequency ω of fundamental mode is

$$\omega = \pi^2(1/a^2 + 1/b^2)\sqrt{\frac{E \cdot t^3}{12(1 - \nu^2)\rho}} \tag{18}$$

where E is the Young's modulus, t is the thickness, ρ is the density, and ν is the

Keeping in view the above relation, we can simulate the value of z for which $I \geqslant I_c$. The simulated value will be equal to RL

4.2 Probability of Landing

The probability of landing of VT-shell at distance r from the surface of the aircraft can be estimated as

$$PLD(r) = \frac{1}{2\pi\sigma_s\sigma_t} \iint_{Sr} \exp\left\{-1/2\left(\frac{s^2}{\sigma_s^2} + \frac{t^2}{\sigma_t^2}\right)\right\} ds\, dt \tag{19}$$

where S_r is the projected region of the vicinity shell over the D-plane (as defined earlier), and σ_s, σ_t are the system errors of the firing gun in azimuth and the elevation planes.

4.3 Probability of Fuze-Functioning

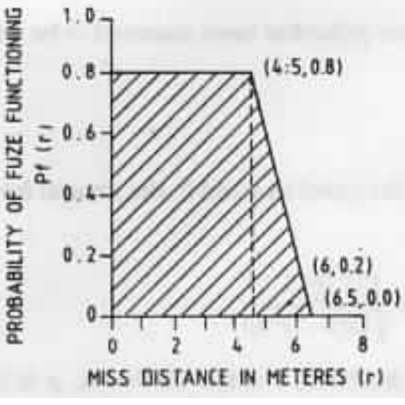
The probability of fuze-functioning at a miss distance 4.5 m is 0.8 and it decreases rapidly with the increase of distance, such that at a distance 6 m, it is 0.2 and 6.5 m it can be treated as 0. Probability of fuze-functioning can be defined as

$$Pdf(r) = \frac{1}{C} Pf(r)$$

where $C = \int_0^{6.5} Pf(r)dr$, and

$$\begin{aligned} Pf(r) &= 0.8 \text{ for } r \leq 4.5 \\ &= -0.4r + 2.6 \quad 4.5 < r \leq 6 \\ &= -0.4r + 2.6 \quad 6 < r \leq 6.5 \\ &= 0.0 \qquad r \geq 6.5 \end{aligned} \tag{20}$$

The probability distribution is shown in the Fig. 3.



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where E is the Young's modulus, t is the thickness, ρ is the density, and ν is the

4.4 Probability that at least k Number of Fragments will Penetrate

Probability that at least k number of fragments of each of mass $P_{km} \geq m$ will penetrate the component, if the VT-shell bursts in the vicinity shell at a distance r from the aircraft's surface is given by[1]

$$P_{km}(r) = 1 - \sum_{N=0}^{k-1} \frac{(M_r)N}{N!} e^{-M_r} \tag{21}$$

where M_r is the average number of fragments penetrating the component. If the VT-shell burst in a vicinity shell at a distance r, then M_r is given by

$$M_r = 0.5 \left[\frac{1}{N_p} \sum_{k=1}^{N_p} N^k \right] \tag{22}$$

where N_p is the total number of points in the vicinity shell and N^k is the number of fragment hits to a component with impact velocity greater than $(V50)_\theta$ if the shell explodes at k- th point of the vicinity shell at a distance r from the surface of the aircraft. $(V50)_\theta$ is shown in Fig. 4, and N^k is evaluated in section 4.5. The factor 0.5 in Eqn (22) is used because of the definition of $(V50)_\theta$.

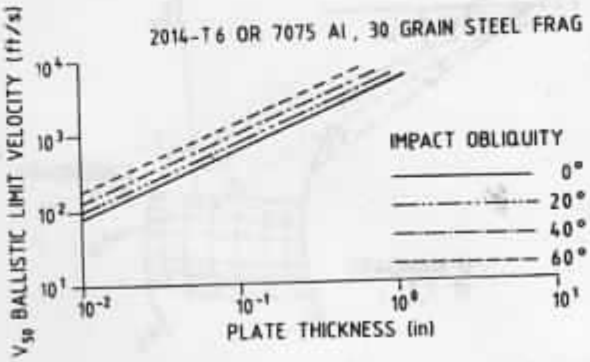

Figure 4. Typical V50-ballistic limits for aircraft strctural materials[7].

4.5 Expected Number of Fragment Hits

Let the shell burst at a point p^k in the vicinity shell of the aircraft. The fragments of the shell moves in the conical angular zones with respect to the axis of the VT-shell. Let there be n_z such uniform conical zones, uniform in the sense that the ejection of the fragments per unit solid angle is the same within a particular zone. Size of VT-shell is very small as compared to that of the aircraft, therefore it can be assumed that the fragments are ejecting as if they are coming from the centre of the shell.

We define $z^k_{i,i+1}$, the zone which is the intersection of two solid cones, with vertex at a point p^k and the interaction of two solid cones whose slant surfaces make angles θ_i, θ_{i+1} respectively with the axis of the shell. And let $n^k_{i,i+1}$, be the total

Fragments per unit solid angle in the $z^{k}_{i,\,i+1}$th angular zone can be given as

$$\int_{i,i+1}^{k} = \frac{n^{k}_{i,i+1}}{2\pi(\cos\alpha'_i - \cos\alpha'_{i+1})} \tag{23}$$

where a_i, a_{i+1}, are explained in Eqn (27).

Let $\omega^{k}_{i,\,i+1}$ be the solid angle subtended by the component in the $z^{k}_{i,\,i+1}$ angular zone and $f^{k}_{i,\,i+1}$ is the fragment density therein, then the total number of fragment hits to the component is given by

$$N^{k} = \sum_{i=1}^{n_z} \omega^{k}_{i,i+1} \cdot \int_{i,i+1}^{k} \tag{24}$$

4. 6 Solid Angle Subtended by a Component in an Angular Zone

The solid angle subtended in the angular zone $z_{i,i+1}$ (the results of this section are independent of k and are true for all values of k) by a component at the centre of gravity (CG) of the shell is determined by the intersecting surface of the component and the angular zone $z_{i,\,i+1}$ mathematically (Fig. 5).

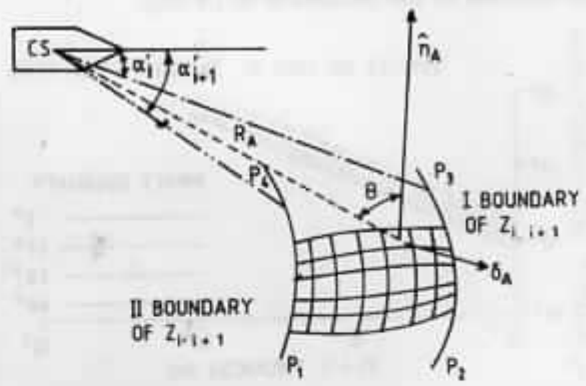

Figure 5. Scenario of solid angle subtended in an angular zone.

$$w_{i,i+1} = \sum_{A_{i,i+1}} \delta w$$

$$\delta w = \frac{|\cos\theta|\,\delta A}{R_A^2} \tag{25}$$

where $A_{i,i+1}$ is the intersecting surface of the component and the zone $z_{i,\,i+1}$ which will differ in stationary and dynamic cases; δA is the small area on the surface $A_{i,i+1}$; R_A is the distance between CG of the shell and the mid-point of δA; and θ is the angle between R_A and normal to the surface at the mid-point of δA.

Value of $\delta\omega$ is evaluated in Eqn (32). Following is the example to evaluate the solid angle subtented by a component of the aircraft in the different angular zones of the aircraft. Similar method can be developed to any component of the aircraft having well defined surface.

Let the VT-fuzed shell burst at a point C_s in the vicinity region of the aircraft, say at time $t = 0$. At the time t, let the coordinates of the CG of the VT-shell be (x_s, y_s, z_s) and the velocity of the shell is 'V_s' in the direction (l_s, m_s, n_s) which is also the direction of its axis, with respect to Frame-I which is fixed in space.

Further let at the time of burst, (x_a, y_a, z_a), be the coordinates of the centre of the aircraft which is also the origin of the Frame-II and let (l_i, m_i, n_i), $i = 1$ to 3 be the direction cosines of the aircraft's axes (i.e., axes of the Frame-II) with respect to Frame-I and this aircraft (Frame-II) is moving with velocity V_a in the direction (l_v, m_v, n_v) in Frame-I.

Let the coordinates of the CG of the shell at the time of burst ($t = 0$) be (x_s, y_s, z_s), and (u_s, v_s, w_s) with respect to the two frames of reference. Transformation from one system of coordinates to other is given as

$$\begin{array}{l|l} x_s = u_s \cdot l_1 + v_s \cdot l_2 + w_s \cdot l_3 + x_a & u_s = x_s \cdot l_1 + y_s \cdot m_1 + z_s \cdot n_1 - x_a \\ y_s = u_s \cdot m_1 + v_s \cdot m_2 + w_s \cdot m_3 + y_a & v_s = x_s \cdot l_2 + y_s \cdot m_2 + z_s \cdot n_2 - y_a \\ z_s = u_s \cdot n_1 + v_s \cdot n_2 + w_s \cdot n_3 + z_a & w_s = x_s \cdot l_3 + y_s \cdot m_3 + z_s \cdot l_3 - z_a \end{array} \qquad (26)$$

Let us assume that VT-shell bursts in stationary position with reference to Frame-I and α_i, α_{i+1}, are the angles which the boundaries of the conical angular zone of fragments $z_{i,i+1}$ make with the positive direction of the shell axis and VF_i, VF_{i+1} are the corresponding velocities of the fragments of these boundaries.

When shell bursts in a dynamic mode, the directions and velocities of fragments, as observed in a stationary frame will be

$$\alpha_i' = \tan^{-1}(V_2/V_1)$$

$$VF_i' = (V_1^2 + V_2^2)^{1/2}$$

where

$$V_1 = V_s + VF_i \cos(\alpha_i)$$

$$V_2 = VF_i \cdot \sin(\alpha_i) \qquad (27)$$

Fragments emerging from C_s, in an angular zone $z_{i,i+1}$ will be confined in a cone making angles α'_i and α_{i+1} respectively with the axis of the shell. Intersection of this cone with the surface of the aircraft is say an area P_1, P_2, P_3, and P_4. Divide the surface enveloping P_1, P_2, P_3, and P_4 into a finite number of rectangular areas $\delta A = \delta l . \delta b$ (say) where δl and δb are dimensions of the rectangular element (Fig. 5).

If point P, whose coordinates with respect to Frame-II are (u_p, v_p, w_p), is the middle point of area δA, then solid angle of area δA subtended at the centre of the

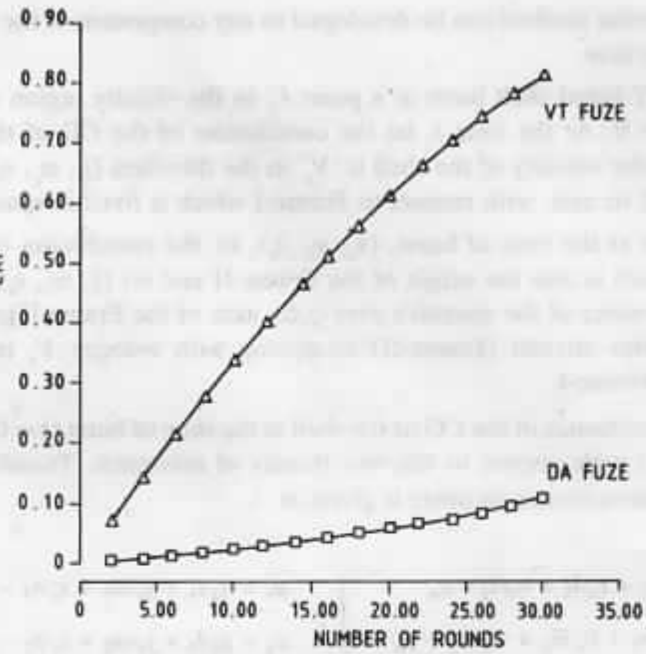

Figure 6. Number of rounds vs CKP for DA- and VT-fuzed ammunition.

Coordinates at point P_t at any time after brust, with respect to a fixed frame are

$$x_{pt} = x_p + V_a \cdot l_v \cdot t; \quad y_{pt} = y_p + V_a \cdot m_v \cdot t; \quad \text{and} \quad z_{pt} = z_p + V_a \cdot n_v \cdot t \tag{28}$$

Where V_a is the velocity of the aircraft and (l_v, m_v, n_v) are direction consines of velocity vector with reference to Frame-I. If ϕ is the angle between shell axis and line $C_s P_t$ where P_t is the position of point P at time t, then first step is to determine the angular zone a'_i, a'_{i+1} in which ϕ lies.

Fragment may come to the point P_t from angular zone $z_{i,i+1}$ with velocity VF'_i, VF'_{i+1} depending upon ϕ is close to a'_i or a'_{i+1}.

Distance travelled by the fragments along the line $C_s - P_t$ in time t, is

$$D_f = VF' \cdot t$$

where $VF' =$ selected (VF'_{i}, VF'_{i+1}) (29)

In Eqn (29), value of VF' is selected from VF' and VF'_{i+1} depending that ϕ is nearer to a'_i or a'_{i+1}.

Actual distance between point C_s and P_t is

From Eqns (29) and (30) we simulate t such that $D_t = D_s$ for confirmed impact. Velocity of impact of a fragment V_{strike} can be given as

$$V_{strike} = (VF'^2 + V_a^2 - 2VF' \cdot V_a \cos \beta)^{1/2}$$

where β is the angle between the positive direction of aircrafts velocity vector and fragment velocity vector. Figure 4 shows the graphs of velocity of fragment $(V50)_0$ versus the penetration in aircraft structural materials is shown. We define $(V50)_\theta$ as the velocity of fragment hitting the component at an angle θ with the normal to the surface, so that its probability of penetrating the component is 50 per cent.

If θ is the angle of impact, then

$$(V50)_\theta = (V50)_0 / \cos \theta \quad (31)$$

where $(V50)_0$ is the required velocity of impact at zero degree angle of obliquity and can be obtained[8] for various thickness of plates and different kinds of projectiles.

If velocity of impact V_{strike} is greater than $(V50)_\theta$ then the solid angle $\delta\omega$, subtended by the small rectangular element, in the angular zone $z_{i,j+1}$ is given by

$$\delta w = \frac{\delta A \cdot |\cos \theta|}{D_s^2} \quad (32)$$

which is added to the Eqn (25).

5. CUMULATIVE KILL PROBABILITY

As the aircraft is considered to have been divided into y parts, let $P_i(j)$ be the single shot kill probability of a typical vital part due to ith burst of fire, each burst having n rounds. The cumulative kill probability of a typical vital part (say, jth) in N burst of fire can be given as

$$CKP\,(j) = 1 - \prod_{i=1}^{N} [1 - P_i(j)]^n \quad (33)$$

Further the aircraft can be treated as killed if at least one of its vital part is killed. Thus the *CKP* for the aircraft as a whole can be given as.

$$CKP = 1 - \prod_{j=1}^{y} [1 - CKP(j)] \quad (34)$$

6. DATA USED

A typical aircraft was used to validate the model given in the present paper. Data used as input to the model for the aircraft is as follows :

6.1 Target Aircraft

Radius of the fuselage = 0.86 m

Distance of geometric centre of aircraft from frontal section = 7.82 m

Material of the aircraft's skin: strong aluminium alloy

Density of the strong aluminium alloy (ρ) = 2800 kg/m^3

Dynamic yield strength (taken) (σ_y) = 550 × 10^6 Pa

Young's modulus of strong aluminium alloy (E) = 75.0 × 10^9 Pa

Poisson's ratio (ν) = 0.33

Table 1. Vital parts and parameters considered in the study

Parameters	Pilot	Fuel tank	Engine
Distance of vital parts from frontal section (m)	3.30	5.24	10.67
Width of vital parts (m)	1.94	1.77	2.26
Equivalent thickness of duraluminium of vital parts assumed (mm)	12	10	8
Critical energy (in joules) required to kill the vital part[6]	678.0	339.0	1356.0
Estimated numbers of fragments to produce the required energy	1	1	2
Velocity V50 for 0° angle of oblique (Fig. 4) (m/s)	699.1	594.5	487.6

6.2 Weapons

An air defence twin barrel gun with DA/VT-fuzed ammunition is considered for this study, with the following parameters.

System error	:	3 mrad
Firing rate	:	5 rounds/s/gun barrel
Probability of fuze-functioning DA	:	0.99
VT	:	0.8 within distance $r \leq 4.5$ m
		0.2 at distance $r = 6$ m
		0.0 at distance $r \geq 6.5$ m
Time of continous firing of guns	:	3 s
Maximum range of gun	:	5000 m
Minimum range of gun	:	500 m
Maximum detecting range	:	10000 m

7. RESULTS AND DISCUSSION

The model was run for data given above. The aircraft have been considered coming across the gun position at an altitude of 100 m and at a speed 300 m/s. The twin barrel gun starts engaging the target aircraft from the range of 2000 m for a period of 3s.

The number of fragments required to defeat a vital part of an aircraft is calculated on the basis of energy criteria[6]. Tables 2 and 3 give the kill probability of various vital parts and cumulative kill probability *(CKP)* of the aircraft as a whole. The results so obtained for the typical aircraft have been presented in Fig. 6 for DA- and VT-fuzed

Table 2. Number of rounds vs CKP of the various vital parts and aircraft as a whole for a typical aircraft due to DA-fuzed ammunition

No. of rounds	Aircraft	Pilot	Fuel tank	Engine
2	.0045	.0015	.0013	.0017
4	.0092	.0030	.0027	.0035
6	.0142	.0047	.0042	.0054
8	.0195	.0064	.0058	.0074
10	.0251	.0083	.0075	.0095
12	.0311	.0103	.0093	.0118
14	.0374	.0124	.0113	.0142
16	.0441	.0146	.0133	.0168
18	.0514	.0171	.0156	.0196
20	.0592	.0197	.0180	.0227
22	.0675	.0226	.0206	.0259
24	.0766	.0257	.0234	.0295
26	.0863	.0291	.0265	.0333
28	.0968	.0327	.0298	.0375
30	.1082	.0368	.0335	.0421

Table 3. Number of rounds vs CKP of the various vital parts and aircraft as a whole for a typical aircraft due to VT-fuzed ammunition

No. of rounds	Aircraft	Pilot	Fuel tank	Engine
2	.0720	.0319	.0302	.0116
4	.1410	.0639	.0604	.0233
6	.2089	.0970	.0919	.0353
8	.2745	.1304	.0241	.0476
10	.3376	.1640	.1568	.0604
12	.3967	.1972	.1891	.0733
14	.4544	.2318	.2224	.0867
16	.5094	.2665	.2562	.1007
18	.5611	.3014	.2900	.1152
20	.6105	.3369	.3245	.1304
22	.6579	.3738	.3600	.1463
24	.7019	.4108	.3960	.1624
26	.7425	.4476	.4323	.1788
28	.7798	.4846	.4686	.1959
30	.8139	.5215	.5055	.2136

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APPENDIX

Let (x_o, y_o) be the centre of the D-plan forming a right handed system of axis, *S*-axis, *T*-axis and *OG*-axis as shown in Fig. 1(a). The line *GO* is perpendicular to the *S*, *T* plane with direction cosines $(-l_0, -m_0, -n_0)$ where

$$l_0 = \cos A \cdot \cos E = \frac{x_0}{|GO|}$$

$$m_0 = \sin A \cdot \cos E = \frac{y_0}{|GO|}$$

$$n_0 = \sin E = \frac{z_0}{|GO|} \quad (1)$$

S-axis which will lie in the so-called azimuth plane will be normal to the elevation

Equation of the plane *GOO'* is

$$l_s x + m_s y + n_s z = 0 \tag{2}$$

Since this plane passes through the three points (0, 0, 0), (x_o, y_o, z_o), and $(x_o, y_o, 0)$, we have,

$$x_0 l_s + m_0 y_s = 0 \tag{3}$$

$$x_0 l_s + m_0 y_s + n_0 z_s = 0 \tag{4}$$

Solving Eqns (3) & (4) along with

$$l_s^2 + m_s^2 + n_s^2 = 1 \tag{5}$$

One gets the direction cosines of *OS*-axis as

$$\left(\frac{m_0}{\sqrt{1-n_0^2}}, \frac{-l_0}{\sqrt{1-n_0^2}}, 0\right) \tag{6}$$

Similarly we get the directions cosines of *OT*-axis as

$$\left(\frac{-n_0 l_0}{\sqrt{1-n_0^2}}, \frac{m_0 n_0}{\sqrt{1-n_0^2}}, \sqrt{1-n_0^2}\right) \tag{7}$$