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A NOTE ON SPHERICAL SHOCK WAVES IN WATER

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Propagation of spherical shock waves produced by an explosion in homogeneous water is discussed by using Witham's Rule. An analytical expression for the variation of density behind the shock front is derived.

INTRODUCTION

Problem of propagation of shock waves in water is of immense importance in Navy. In an earlier paper, Singh and Bola (1972) discussed propagation of shock waves in non-homogeneous water, where, gravitational force was considered. In that paper authors used energy equation to solve Rankine-Hugoniot equation. They used Whitham's rule to obtain fluid parameters behind the shock.

In the present paper we use modified Tait's equation of state instead of energy equation. Since it is well known that Tait's equation of state holds infront as well as behind the shock, this can be used as one of the jump conditions. Using Whitham's rule we obtain a differential equation between density ratio and the distance of shock from the point of explosion. This equation is integrated analytically. Thus knowing density ratio as a function of shock radius, other fluid parameters are also found and are shown in Table 1.

BASIC FORMULATIONS

We assume that water column is homogeneous and pressure throughout is one bar, i. e. the atmospheric pressure. It is assumed that no body force is acting on the medium. Thus equation of state of water is

$$p_2=p_1+A(s)\left[\left(\frac{\rho_2}{\rho_1}\right)^n-1\right] \tag{1}$$

where p, ρ are pressure and density, subscripts 1 and 2 denote value ahead and behind the shock respectively and $A(S)=2.941$ K bars.

Let us assume that a spherical charge of *RDX/TNT* of radius 3.75 cm is fired below the water surface at large depth. *A* spherical shock will propagate in all the directions. Taking point of explosion as origin, let *R* be the radius of shock at any time t. The jump conditions across the shocks at $r=R$ are

$$u_2=\left[\frac{A}{\rho_1}\{(\rho_2/\rho_1)^n-1\}(1-\rho_1/\rho_2)\right]^{1/2} \tag{2}$$

$$U=\left[\frac{A\{(\rho_2/\rho_1)^n-1\}}{\rho_1(1-\rho_1/\rho_2)}\right]^{1/2} \tag{3}$$

$$E_2=E_1+\tfrac{1}{2}[2\,p_1+A\{(\rho_2/\rho_1)^n-1\}]\left(\frac{1}{\rho_1}-\frac{1}{\rho_2}\right) \tag{4}$$

where the symbols have usual meaning (Singh and Bola 1972). These are the three jump conditions in five unknown parameters P_2, ρ_2, u_2, U and E_2. These are supplemented by eqn. (1). We require one more relation to solve them explicitly in terms of shock radius. As before (Singh and Bola 1972) we use Whitham's rule to find this extra relation.

Discussion of the Problem

It is known Whitham's Rule is applicable for the case of flow in a non-uniform channel. If we take cross-sectional area of channel $4\pi r^2$, we can generalise this problem for the case of spherical shocks. Equation of motion along positive characteristic axis $dR/dt=u_2+c_2$ is

$$dp_2+\rho_2\,c_2\,du_2+\frac{\rho_2\,c_2}{u_2+c_2}\left[\frac{2c_2\,u_2}{R}\right]dR=0. \tag{5}$$

This is an extra relation between p_2, u_2, ρ_2 and R.

If we substitute expressions for p_2, ρ_2, u_2 from relation (1)–(4) we get after simplifications

$$\frac{R}{2}\,\frac{d\delta}{dR}=-\delta^2\,K(\delta) \tag{6}$$

where

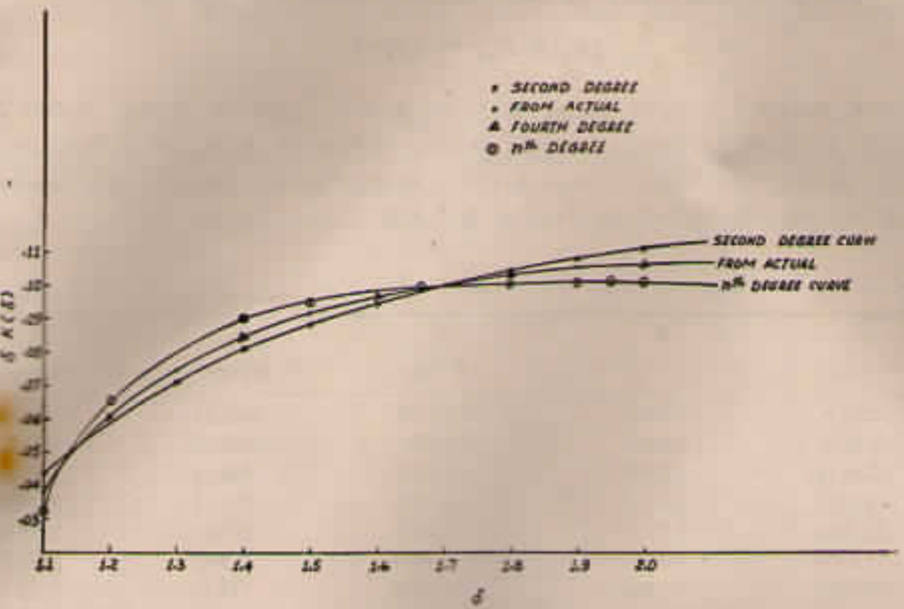
$$\frac{1}{K(\delta)}=\tfrac{1}{2}\delta\left[1+\left\{\frac{n\delta^n}{(\delta^n-1)(\delta-1)}\right\}^{\frac{1}{2}}\right]\times \left[1+\tfrac{1}{2}\sqrt{\frac{n\delta^n(\delta-1)}{(\delta^n-1)}}+\tfrac{1}{2}\sqrt{\frac{\delta^n-1}{n\delta^n(\delta-1)}}\right] \tag{7}$$

where $\delta=\rho_2/\rho_1$.

Equation (6) is a differential equation between δ and R. It is not possible to integrate this equation as it is. Variation of $K(\delta)$ versus δ is shown in Fig. 1. We have fitted various types of curves in (7) and it is found that best fit curve is of the form

$$K(\delta)=a+b/\delta^4 \tag{8}$$

where $a=0.11316$, $b=-0.10997$,



Fig, 1. Variation of different curves fitted in data relating $\delta KI(\delta)$ and δ.

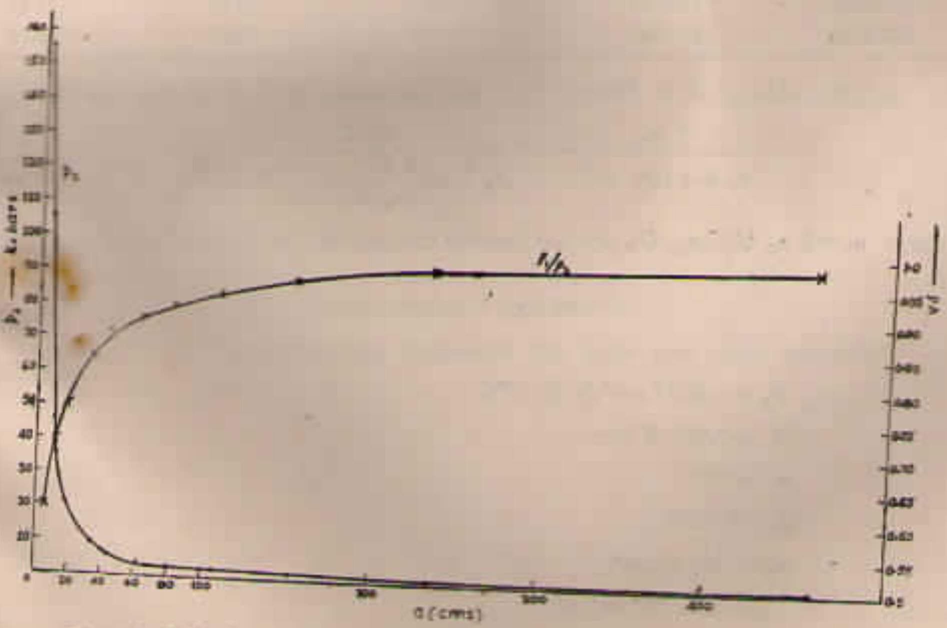


Fig. 2. Variation of shock pressure and densities jump with distance R.

Using this expression for $K(\delta)$, eqn. (6) is integrated in the form

$$\frac{\rho_2}{\rho_1} = \frac{1}{a^{1/4}}\left[K_*(R/R_*)^{-8a} - b\right]^{1/4} \tag{9}$$

where $K_* = a + b\,\delta_*^{\,4}$. δ_* is initial value of density jump at charge radius R_*. This is an analytic form of density ratio. Knowing ρ_2/ρ_1 as a function of shock radius, variations of p_2, u_2, U can be found. Results of numerical computations are shown in Table I and Fig. 2.

TABLE I

R cms	δ	p_2 K ber	u_2 m/sec	U m/sec
3.75	1.7465	165.0146	2648.35	6196.44
5.0541	1.6667	116.594	2154.66	5386.65
11.54325	1.42857	36.1032	1040.19	3467.30
17.5212	1.3333	20.7353	719.55	2880.54
22.5906	1.25	11.8885	487.01	2435.03
32.8338	1.21951	9.4545	413.12	2295.13
39.7669	1.19045	7.4718	344.35	2153.54
42.6534	1.16279	5.8359	284.45	2037.58
62.1572	1.13636	4.4951	236.25	1968.79
80.1956	1.11111	3.3704	182.10	1821.06
107.3625	1.00695	2.4440	139.84	1748.05
152.2980	1.06383	1.638	98.94	1649.06
238.9665	1.04167	1.0235	63.73	1593.25
465.4125	1.02041	0.4558	30.06	1503.21

Initial value of δ is found from (see *Buchanan et al.* 1959) the relation

$$\frac{2\,p_D}{p_1 + A\,(\delta_*^{\,n} - 1)} = \frac{\rho_D\,U_D}{\rho_1\left\{\dfrac{A\,(\delta_*^{\,n} - 1)\,\delta_*}{\rho_1\,(\delta_* - 1)}\right\}^{\frac{1}{2}}} + 1 \tag{10}$$

where $p_D = \frac{1}{4}\,\rho_D\,U_D^2$. ρ_D, U_D are the density and velocity of detonation of explosive.

NUMERICAL COMPUTATIONS

Following data was ased for numerical computation.

$V_f = 1.0027\ cm^3/g$ at 24°C

$A = 2.941$ K bars

$n = 7.25$

$\delta_* = 1.7465$

$\rho_D = 1.68\ g/cm^3$

$U_D = 7.8 \times 10^3\ m/sec.$

In the present analysis we have neglected the effect of gas bubble on the shock front. Results of numerical computation are shown in Table I and Fig. 2

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